INTRODUCTION
Damping ratio in shear ($D_s$) is an important soil property for the geophysical characterisation of sediments, seismic design of civil engineering facilities and study of energy dissipation in geomaterials (Richart et al., 1970; Cascante & Santamaria, 1996; Ishihara, 1996). At small strains, damping ratio reaches a minimum value, denoted by $D_s$min. Mechanisms of energy loss at small strains are not well understood in the literature. Santamaria & Cascante (1996) reported that processes other than frictional losses are involved in the dissipation of energy at small strains. Meng (2003) and Senetakis et al. (2012) related $D_s$min to the mean effective confining pressure and gradation characteristics of the soil. Senetakis et al. (2013) discussed the influence of particle morphology on $D_s$min and attributed it to the shape descriptors of the particles. The effect of particle shape on small-strain dynamic properties has also been emphasised by Santamaria & Cascante (1998), Cho et al. (2006) and Payan et al. (2016). Nevertheless, no systematic investigation of the effect of particle shape on the damping properties of sands has to date been reported in the literature.

The main objective of this note is to report on the results of an experimental study on the effect of particle shape on the small-strain damping ratio of dry sands. For this purpose, samples of sands with a variety of grain shapes are examined in a resonant column apparatus in torsional mode of vibration to determine their small-strain damping ratio using two different approaches: free-vibration decay and half-power bandwidth methods. The analyses of the results are incorporated into a new expression for the prediction of small-strain damping ratio of dry sands.

TEST MATERIALS AND METHODS
Eleven sands with various gradations and particle shapes were tested in this study. The grading curves of the sands are presented in Table 1. The grain size characteristics as well as particle shape descriptors are given in Table 1. Blue and uniform Sydney sands were used for independent verification purposes and were not included in the model development. All test soils were classified as SP according to the Unified Soil Classification System (USCS) with a coefficient of curvature ($C_s$) close to unity. Dry samples were prepared to target void ratios in a metal split mould placed directly on the base pedestal of a resonant column apparatus (RCA) with fixed-free ends. For the data analysis, 19 samples were prepared and tested in the RCA under isotropic confining pressures, $p'$, ranging from 50 to 800 kPa. Samples’ initial void ratios, $e_o$, are given in Table 1. All the specimens were tested in a dry state in the torsional mode of vibration. The sequence of increasing $p'$ adopted in the tests was 50, 100, 200, 400, 600 and 800 kPa.

The particle shape descriptors in Table 1 were quantified visually in an optical microscope adopting a widely used empirical chart proposed by Krumbel & Sloss (1963) (Fig. 2). For a given sand, 30 particles were randomly selected and two shape descriptors, namely roundness, $R$, and sphericity, $S$, were quantified. The roundness is related to the local surface features of the sand particles, and is defined as the ratio of the average radius of the surface features to the radius of the largest sphere inscribed in the sand particle. Sphericity is an indication of the general shape of sand particles and is quantified as the ratio of the radius of the largest circumscribed sphere to the particle. The definitions of $R$ and $S$ are shown schematically in Fig. 2. Only the mean values of $R$ and $S$ denoted by $R$ (mean) and $S$ (mean) are presented in Table 1. The regularity, $p_o$, was calculated as the algebraic mean of the roundness and sphericity, that is, $p_o = 0.5 \times (R + S)$. Regularity was introduced by Cho et al. (2006) to simultaneously account for the effects of both roundness and sphericity in the mechanical behaviour of geomaterials. The mean value of regularity for each sample, denoted by $p$ (mean), was used in the analyses in this study.

The values of small-strain damping ratio in shear were obtained using the free-vibration decay (FVD) method (ASTM, 1955) as well as the half-power bandwidth (HPB) method. In the FVD method, three successive cycles during free vibration of the samples were adopted for small-strain damping ratio calculations, as suggested by Stokoe et al. (1999). A typical example of the experimental results along with the calculations to obtain the small-strain damping ratio using FVD and HPB methods are given in Figs 3 and 4, respectively. Note that the measurements of small-strain damping ratio in this study...
Fig. 1. Particle size distribution curves of the tested sands

Table 1. Different properties of the soils tested in the study

<table>
<thead>
<tr>
<th>Laboratory material (sand)</th>
<th>Grading</th>
<th>Particle shape descriptors*</th>
<th>$e_0$†</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d_{50}$‡</td>
<td>$C_u$‡</td>
<td>$R$ (mean)</td>
</tr>
<tr>
<td>Sydney</td>
<td>0·31</td>
<td>1·95</td>
<td>0·61</td>
</tr>
<tr>
<td>Bricky</td>
<td>0·47</td>
<td>2·19</td>
<td>0·48</td>
</tr>
<tr>
<td>White (Blue circle)</td>
<td>0·24</td>
<td>1·69</td>
<td>0·71</td>
</tr>
<tr>
<td>Newcastle</td>
<td>0·33</td>
<td>1·94</td>
<td>0·64</td>
</tr>
<tr>
<td>Nepean (River)</td>
<td>0·59</td>
<td>4·15</td>
<td>0·55</td>
</tr>
<tr>
<td>Uniform Sydney</td>
<td>0·36</td>
<td>1·18</td>
<td>0·61</td>
</tr>
<tr>
<td>Blue</td>
<td>1·88</td>
<td>4·11</td>
<td>0·24</td>
</tr>
<tr>
<td>Uniform Blue</td>
<td>0·69</td>
<td>1·99</td>
<td>0·24</td>
</tr>
<tr>
<td>50% uniform bricky, 50% uniform Blue</td>
<td>0·54</td>
<td>1·96</td>
<td>0·36</td>
</tr>
<tr>
<td>70% uniform bricky, 30% uniform Blue</td>
<td>0·49</td>
<td>2·01</td>
<td>0·41</td>
</tr>
<tr>
<td>30% uniform bricky, 70% uniform Blue</td>
<td>0·59</td>
<td>1·99</td>
<td>0·31</td>
</tr>
</tbody>
</table>

*Obtained according to the modified version of particle shape characterisation chart developed by Cho et al. (2006).
†$e_0$, initial void ratio.
‡$d_{50}$, mean grain size; $C_u = d_{60}/d_{10}$.

$R$, roundness; $S$, sphericity; $\rho$, regularity.

Fig. 2. Particle shape characterisation chart (Krumbein & Sloss, 1963; Cho et al., 2006)
corresponded to shear strain amplitudes less than $10^{-3}\%$ and thus the behaviour falls within the linear-elastic range (Oztoprak & Bolton, 2013). Comparisons between the results from FVD and HPB methods were conducted for all the tests and the results are illustrated in Fig. 5. Within the scatter of the data, the two methods provide reasonably similar small-strain damping ratio values, which is in agreement with the recent work by Senetakis et al. (2015). Small-strain damping ratios obtained using the FVD method were used for the model development and verification in this study.

RESULTS AND DISCUSSION

Typical test results in terms of the variation of small-strain damping ratio, $D_{s\text{,min}}$, plotted against the effective confining pressure, $p'$, normalised with respect to the atmospheric pressure, $p_a$, for different particle shapes, but similar initial void ratios, are presented in Fig. 6. As previously observed by Menq (2003) and Senetakis et al. (2012, 2013), among others, the results show $D_{s\text{,min}}$ decreases with increasing isotropic confining pressure. However, more importantly, they show a strong dependency of $D_{s\text{,min}}$ on the shape descriptor $\rho$, particularly at the lower ratios of $p'/p_a$.

In order to isolate more clearly the influence of particle shape on small-strain damping ratio, the effects of gradation must be excluded from the observed trends in Fig. 6. To this end, the authors note that the small-strain damping ratio is not influenced by the soil density or the void ratio (Santamarina & Cascante, 1998), and that $D_{s\text{,min}}$ is related to $p'/p_a$ in the form of a power law as (Menq, 2003; Senetakis et al., 2012, 2013)

$$D_{s\text{,min}} = C \left(\frac{p'}{p_a}\right)^x$$

$$x = 1 - 20\%$$

$$f_1 = 62.15 \text{ Hz}$$

$$f_2 = 64.26 \text{ Hz}$$

$$f_n = 62.90 \text{ Hz}$$

$$f_2 = 64.26 \text{ Hz}$$

$$f_1 = 62.15 \text{ Hz}$$

$$f_n = 62.90 \text{ Hz}$$

$$D_{s\text{,min}}(\text{FVD method})$$

$$D_{s\text{,min}}(\text{HPB method})$$

$$\rho$$

$$p_a$$

$$p'$$

$$C$$

$$x$$

$$D_{s\text{,min}}$$

$$\rho$$

$$p_a$$

$$p'$$

Fig. 3. Typical test results and calculations based on FVD method for the estimation of small-strain damping ratio; dry Sydney sand; $e_o = 0.8$, $p' = 100$ kPa

Fig. 4. Typical test results and calculations based on HPB method for the estimation of small-strain damping ratio; dry Sydney sand; $e_o = 0.8$, $p' = 100$ kPa

Fig. 5. Comparison between FVD and HPB methods for calculations of small-strain damping ratio

Fig. 6. Typical test results and calculations based on FVD method for the estimation of small-strain damping ratio; dry Sydney sand; $e_o = 0.8$, $p' = 100$ kPa

Fig. 7. Typical test results and calculations based on HPB method for the estimation of small-strain damping ratio; dry Sydney sand; $e_o = 0.8$, $p' = 100$ kPa

Fig. 8. Comparison between FVD and HPB methods for calculations of small-strain damping ratio

Fig. 9. Typical test results and calculations based on FVD method for the estimation of small-strain damping ratio; dry Sydney sand; $e_o = 0.8$, $p' = 100$ kPa

Fig. 10. Typical test results and calculations based on HPB method for the estimation of small-strain damping ratio; dry Sydney sand; $e_o = 0.8$, $p' = 100$ kPa

Fig. 11. Comparison between FVD and HPB methods for calculations of small-strain damping ratio

Fig. 12. Typical test results and calculations based on FVD method for the estimation of small-strain damping ratio; dry Sydney sand; $e_o = 0.8$, $p' = 100$ kPa

Fig. 13. Typical test results and calculations based on HPB method for the estimation of small-strain damping ratio; dry Sydney sand; $e_o = 0.8$, $p' = 100$ kPa

Fig. 14. Comparison between FVD and HPB methods for calculations of small-strain damping ratio

Fig. 15. Typical test results and calculations based on FVD method for the estimation of small-strain damping ratio; dry Sydney sand; $e_o = 0.8$, $p' = 100$ kPa

Fig. 16. Typical test results and calculations based on HPB method for the estimation of small-strain damping ratio; dry Sydney sand; $e_o = 0.8$, $p' = 100$ kPa

Fig. 17. Comparison between FVD and HPB methods for calculations of small-strain damping ratio

Fig. 18. Typical test results and calculations based on FVD method for the estimation of small-strain damping ratio; dry Sydney sand; $e_o = 0.8$, $p' = 100$ kPa

Fig. 19. Typical test results and calculations based on HPB method for the estimation of small-strain damping ratio; dry Sydney sand; $e_o = 0.8$, $p' = 100$ kPa

Fig. 20. Comparison between FVD and HPB methods for calculations of small-strain damping ratio

Fig. 21. Typical test results and calculations based on FVD method for the estimation of small-strain damping ratio; dry Sydney sand; $e_o = 0.8$, $p' = 100$ kPa

Fig. 22. Typical test results and calculations based on HPB method for the estimation of small-strain damping ratio; dry Sydney sand; $e_o = 0.8$, $p' = 100$ kPa

Fig. 23. Comparison between FVD and HPB methods for calculations of small-strain damping ratio
where \( C \) and \( \kappa \) are material parameters, which a priori may be considered functions of gradation and particle shape of the soil. However, based on the work of Menq (2003), and Senetakis et al. (2012, 2013), exponent \( \kappa \) is not influenced by gradation. Therefore, it is assumed in this work that \( \kappa \) is only a function of particle shape, but \( C \) is affected by both grain size characteristics and particle shape as

\[
C = C_1(\text{grain size characteristics}) \times C_2(\text{particle shape})
\]  

(2)
in which $C_1$ captures the effect of gradation and $C_2$ is a function of particle shape. Menq (2003) proposed the following expression for $C_1$

$$C_1 = 0.55 \times C_u^{0.1} \times d_{50}^{-0.3}$$  

(3)

where $C_u$ and $d_{50}$ are the coefficient of uniformity and the mean grain size (in mm), respectively. Adopting equation (1), parameters $C$ and $\kappa$ can be obtained for each test soil from the best fits to the experimental data shown in Fig. 6. $C_2$ can then be extracted from $C$ using equations (2) and (3).

Variations of $C_2$ and $\kappa$ for the test soils against the shape descriptors $R$, $S$ and $\rho$ are shown in Figs 7 and 8. For both parameters, within the scatter of data, the systematic effect of particle shape can be observed. Linear best fits to the experimental data (using the minimum least-square error

$$\kappa = 0.55 R - 0.71$$

$r^2 = 0.52$

(a)

$$\kappa = 0.95 S - 1.09$$

$r^2 = 0.67$

(b)

$$\kappa = 0.72 \rho - 0.86$$

$r^2 = 0.61$

(c)

Fig. 8. Variation of $\kappa$ with different shape descriptors: (a) roundness; (b) sphericity; (c) regularity

$$D_{s,min,measured} = D_{s,min,predicted}$$

$+/–20\%$

Fig. 9. Comparison between measured and predicted values of small-strain damping ratio
method) along with the corresponding coefficients of correlation ($r^2$) are also depicted in Figs 7 and 8. As can be seen, the absolute value of exponent $\kappa$ decreases with an increase in regularity. This observation is in quantitative agreement with the work by Senetakis et al. (2012) on dry granular soils, implying that the effect of confining pressure on $D_{\text{s,min}}$ becomes less prominent for sands with increasing value of regularity, $p$. Similarly, parameter $C_2$ decreases as regularity increases, an aspect which has been neglected in the previous studies of small-strain damping ratio.

A NEW MODEL OF $D_{\text{s,min}}$ INCLUDING PARTICLE SHAPE

Considering the similar effects of $S$ and $R$ on $C_2$ and $\kappa$ (Figs 7 and 8), regularity, $p$, is deemed to be an effective parameter to incorporate the effect of particle shape on small-strain damping ratio in this study. Thus, based on the best fits in Figs 7(c) and 8(c), and equations (1)–(3), an expression for small-strain damping ratio of sands including the effects of particle shape and gradation may be proposed as follows

$$D_{\text{s,min}} = (0.55 \times C_u^{0.1} \times d_{50}^{-0.3}) \times (-2.06 + 2.43)$$

To explore the validity of equation (4), measured against predicted values of $D_{\text{s,min}}$ are plotted in Fig. 9 for three independent sets of tests: uniform Sydney sand with two initial void ratios of 0·75 and 0·85, and Blue sand with the initial void ratio of 0·75. Also presented in Fig. 9 are the damping ratio data reported by Senetakis et al. (2012) plotted against the predicted values from equation (4). For the data by Senetakis et al. (2012), regularity ($\rho$) values of 0·5 and 0·7 were reported for crushed sand and river sand, respectively. Considering the difficulties in measuring damping ratio at small strains, and the extensive scatter in the data that is reported in the literature, a very good comparison between measured and predicted values of $D_{\text{s,min}}$ is obtained for all practical purposes.

CONCLUDING REMARKS

A set of resonant column tests has been performed with a focus on the determination of the small-strain damping ratio of dry sands using two different approaches: free-vibration decay and half-power bandwidth methods. Using systematic normalisations, the effect of particle shape is isolated and incorporated into the development of a new expression for determination of small-strain damping ratio for sands subject to isotropic confining stress. Particle shape is expressed by means of regularity, which is the average of two particle shape descriptors: the roundness and the sphericity. Comparisons of predicted values of small-strain damping ratio based on the new model and the data reported in this study, and also in the literature, demonstrate a satisfactory performance of the new model.

NOTATION

- $C_2$: material parameter describing the contribution function of grain size characteristics in small-strain damping ratio of sand
- $D_{\text{equipment}}$: damping of equipment
- $D_s$: damping ratio in shear
- $D_{\text{s,min}}$: small-strain damping ratio in shear
- $D_{\text{s,min}}(\text{specimen})$: measured small-strain damping ratio in shear of specimen
- $D_{\text{s,min}}$ measured: measured small-strain damping ratio in shear of specimen
- $d_{50}$: grain diameter at 10% passing
- $d_{60}$: mean grain size of sand
- $d_{90}$: grain diameter at 60% passing
- $e_0$: initial void ratio
- $f_{s1,s2}$: natural resonant frequency corresponding to 0·707 times the maximum shear strain amplitude of vibration
- $N$: number of inscribed spheres in sand particle
- $p'$: isotropic confining pressure
- $p_a$: atmospheric pressure
- $R$: roundness
- $r_{\text{max-in}}$: radius of largest sphere inscribed in sand particle
- $r_{\text{min-eir}}$: radius of smallest sphere circumscribed to sand particle
- $r_1, r_2, \ldots, r_i$: radius of spheres inscribed in sand particle
- $r^2$: coefficient of correlation
- $Z_{\text{max}}$: maximum shear strain amplitude of vibration
- $\delta$: logarithmic decrement of free vibration
- $\kappa$: material parameter related to small-strain damping ratio as a function of particle shape
- $\rho$: regularity

REFERENCES


Menq, F. Y. (2003). Dynamic properties of sandy and gravelly soils. PhD dissertation, University of Texas, Austin, TX, USA.


