Dynamics of potential fill–backfill material at very small strains

K. Senetakis\textsuperscript{a,\*}, B.N. Madhusudhan\textsuperscript{b}

\textsuperscript{a}University of New South Wales, Sydney, Australia
\textsuperscript{b}University of Southampton, Southampton, UK

Received 24 September 2014; received in revised form 21 April 2015; accepted 22 May 2015
Available online 26 September 2015

Abstract

The paper presents a synthesis of past and recently acquired laboratory test results on granular soils using wave propagation techniques at very small strain amplitudes. Resonant column tests on uniform to well-graded coarse sands and gravels of angular and low sphericity grains were analyzed. Empirical-type equations were developed for the prediction of the elastic modulus and material damping at small strains considering the effects of the grading characteristics, the isotropic effective stress and the void ratio. The $G_0-p'$ relationship, expressed through exponent $n_G$, was affected by the sample preparation method. For the narrow range in relatively low pressures adopted in the study, it was observed that $n_G$ decreased slightly with an increase in relative density. Due to the limited initial void ratios of those tests, the effect of the preparation method was not incorporated into the proposed formulae for the $n_G$ prediction. In this direction, additional experiments from the literature, which adopted the resonant column and bender element methods, were further analyzed, including soils of variable types tested with a wider range in relative densities. By employing typical formulae from the theory of elasticity, the bulk modulus and the changes in void ratio were estimated based on the change in isotropic effective stress in the literature data. Considering the recent micromechanical experimental findings associated with the nature of the contact response of soil particles, the importance of soil type and particle-contact behavior in the constant-state response of soils was demonstrated and quantified. Material damping values ranged from about 1.10% to about 0.45% for $p'$ from 25 to 200 kPa with a slight decrease in $D_{s0}$ with an increase in pressure.

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Keywords: Resonant column; Bender elements; Dry sand; Shear modulus; Damping ratio; Elastic properties

1. Introduction

The resonant column method provides reliable measurements of modulus and damping at very small to medium strains, within a range of about $10^{-4}$ to $10^{-2}$%. Modulus derivations refer to secant stiffness and can provide an excellent indication of fabric effects. Constant-state stiffness and material damping are pressure-dependent and the $G_0-p'$ and $D_{s0}-p'$ relationships are expressed through Eqs. (1) and (2), where $G_0$ and $D_{s0}$ are the small-strain shear modulus and the material damping, respectively, and $p'$ is the isotropic effective stress. The power $n_G$ and $n_D$ in these formulae express the effect of $p'$ in the constant-state properties of soils, whilst $A_G$ and $A_D$ are material-dependent constants (Santamarina and Cascante, 1998; Santamarina et al., 2001). With reference to dry granular soils, material damping at very small strains is not affected significantly by the loading frequency (Menq, 2003); and thus, derivations for energy losses may also be considered in the resonant column method without any considerable effect of viscous damping on dry sands or gravels.

\[
G_0 = A_G \times (p')^{n_G} 
\]
Past and recently acquired research works have demonstrated that resonant column test results may also provide useful information related to the effect of properties at the grain scale on the macro-scale response of soils and a better understanding of the dominant mechanisms in particulate media during cyclic/dynamic loading (e.g., Santamarina and Cascante, 1996, 1998; Cascante and Santamarina, 1996; Senetakis et al., 2013a, 2013b). This is because the resonant column method provides an indication of the fabric effects even in the range of extremely small deformations (e.g., Cascante and Santamarina, 1996), and these fabric effects, from a micromechanical point of view, are linked to the magnitude and the distribution of the contact forces (e.g., Radjai and Wolf, 1998) and the probable preferable concentration of normals in the vertical direction within a granular assembly of particles (e.g., Yoshimine et al., 1998). For example, it has been shown through resonant column tests that due to the plastic nature of the particle contact response, which is more dominant in the sliding direction (e.g., micromechanical experimental findings by Cole and Peters, 2007, 2008, Cole et al., 2010, and Senetakis et al., 2013c), the $G_0-p'$ relationship cannot be described efficiently by the Hertz–Mindlin theory. This theory could predict a value for the exponent $n_G$ equal to $1/3$ (Santamarina and Cascante, 1996), which represents more effectively particulate media with an elastic particle-contact response in nature. However, higher values for $n_G$ have been determined through resonant column tests or derived from other wave propagation techniques, such as the bend element method or cyclic-dynamic triaxial tests (e.g., Hardin and Richart, 1963, Hardin, 1978, Kokusho, 1980, Chung et al., 1984, Tanaka et al., 1987, Goto et al., 1987, Menq, 2003, Cho et al., 2006, Senetakis et al., 2012 among others). This is because of the visco-plastic to brittle nature of the contact response of soil particles (Cascante and Santamarina, 1996).

Through wave propagation experiments, Cho et al. (2006) found a significant effect of particle shape in the $G_0-p'$ relationship. They attributed their observations to the possible effect of the particle contact response which alters between irregular and regular in shape particles, perhaps because of the more pronounced grain crushing or micro-crushing and overall changes in bulk volume when more irregularly shaped particles are considered than regularly shaped particles. Through one-dimensional compression tests on reference particles, Cavarretta et al. (2010) verified the significant effect of particle shape in the overall compression-pressure relationship which, in turn, affects the fabric, and thus, the stiffness of geo-materials. In this direction, Senetakis et al. (2012), who studied the small-strain dynamic properties of fine- to medium-grained sands, reported a significant effect of particle shape on the constants $n_G$ and $A_G$.

Menq (2003) and Menq and Stokoe (2003) noticed in their resonant column experiments a dominant effect of the coefficient of uniformity in the $G_0-p'$ relationship. This trend has been correlated, partly, through numerical simulations and quantification of the grain size distribution effects on isotropically consolidated granular assemblies, to the distribution and magnitude of the particle contact forces (e.g., Radjai and Wolf, 1998, Radjai et al., 1998). Recently, Senetakis et al. (2012, 2013a) highlighted the importance of particle type and morphology in the $G_0-p'$ relationship. Senetakis et al. attributed their observations primarily to the possibly more pronounced damage of surface roughness because of the coupled normal force – deflection and tangential force – deflection responses at particle contacts. These derivations were based on the recent quantification of particle damage by Senetakis et al. (2013c) and measurements of friction and stiffness at particle contacts by Senetakis et al. (2013c, 2013d) and Senetakis and Coop (2014, 2015) on reference strong particles of a quartz sand and reference weak particles of a biogenic crushable sand. For example, Fig. 1 presents the coupled effect of normal load and tangential load – deflection responses at the contacts of two quartz particles before and after sliding tests by Senetakis et al. (2013c). In this figure, a cross-section of a quartz particle is shown within the area of contact and sliding on the surface of another similar particle before and after micromechanical sliding tests. They quantified the damage of the surface roughness using white light interferometry. As demonstrated in the figure, the coupled effect of the normal force and sliding significantly reduced the magnitude of surface roughness.

\[ D_{d0} = A_D \times (p')^{n_D} \]  

Fig. 1. Quantification of surface roughness damage due to shearing between two quartz particles: interferometer section before and after shearing (Senetakis et al., 2013c) – Horizontal size is 141.5 μm.
explained by the signification mechanisms from small to medium strains, may be 
found by Senetakis et al. (2012, 2013a) in their resonant column tests, along with the probable different energy dis-
nipation from the contacts of strong and weak particles with respect to friction and stiffness on the micro-scale level. Previous research works, for example, Sadd et al. (1993, 2000), Sazzad and Suzuki (2011), Barreto and O’ Sullivan (2012) or Huang et al. (2014), have highlighted through numerical studies the partially 
important effect of the particle contact characteristics (e.g., coefficient of friction at particle contacts, stiffness or particle-
contact response model) in the overall macro-scale behavior of soils.

In this study a synthesis of torsional resonant column tests is 
presented on a potential fill–backfill material tested on hard 
grains with particle size from coarse sand to fine gravel and a variety 
of coefficients of uniformity. Particular focus is given to the constant-state properties of the samples in terms of the \( G_{\sigma}'-p' \)
and \( D_{50}-p' \) relationships in the range of relatively low 
confining stresses which represent typical low working loads 
for geotechnical engineering design. For this range of rela-
tively low stresses, no measurable damage to particles was 
observed, at least visually. For further interpretations and a link 
the macro-scale observed behavior of the samples and the 
probable effect of the particle contact response, additional 
dynamic test results that have been published by the authors 
were included and re-analyzed by means of investigating the 
\( G_{\sigma}'-p' \) relationship adopting formulae from the theory of 
elasticity.

2. Materials, sample preparation and testing program

2.1 Primary dynamic testing program: materials and 
experimental techniques

Fifteen samples, denoted as "SAMPLE01" to "SAM-
PLE15", were created in the laboratory from the same parent 
soil of hard, angular particles of low sphericity. The labora-
tory-created samples had a mean grain size, \( d_{50} \), a coefficient 
of uniformity, \( C_u \) and a maximum grain size, \( d_{\text{max}} \) in the 
ranges of 1.33–10.1 mm, 1.03–12.5 and 2.00–12.7 mm, 
respectively. The specific gravity of solids was determined 
by adopting the water pycnometer method (ASTM, 2002, 
D854-02) with a value equal to 2.67, which was found 
independent of the particle size. Adopting the unified classi-
fication system USCS (ASTM, 2000a, D2487-00), the soils 
were classified as SP, SP-SW, GP and GW with varying 
percentages of sand and gravel portions. It is noted that by 
employing the ASTM specifications, the gravel-size grains 
were defined as the fraction retained on sieve No. 4 (size equal 
to 4.75 mm) and the "gravelly" samples for soils that had a 
gravel content of more than 50% of dry mass. These samples, 
presented in Fig. 2 and Table 1, comprised the materials of the 
primary dynamic testing program of the study and they were 
subjected to isotropic torsional resonant column (RC) testing in 
a dry state. The maximum void ratio (\( e_{\text{max}} \)) of the samples 
was determined following the ASTM (2000b) D4254-00 specification, 
whilst the vibratory table method (ASTM, 2000c, D4253-
00) was used to determine the minimum void ratio (\( e_{\text{min}} \)). An 
optical microscope image of the fine-sand fraction of the parent 
soil is given in Fig. 3 (fraction 0.075–0.180 mm). Adopting 
the empirical chart for the quantification of the particle shape 
descriptors by Krumbein and Sloss (1963), it was revealed that 
the particles had very low roundness with \( R \) values between 
0.1 and 0.3 and very low sphericity with \( S \) values between 
0.3 and 0.5 for the majority of the particles. It is noted that \( R \) 
and \( S \) denote the mean roundness and the sphericity of 
the grains, respectively (Krumbein and Sloss, 1963; Santamarina 
et al., 2001; Cho et al., 2006).

For each sample of the primary dynamic testing program, 
two specimens were constructed in a standard split mold, 
approximately 71 mm in internal diameter and 142 mm in 
height, one loose to very loose specimen and one dense to very 
dense specimen. To prepare the loose samples, the hand 
spooning method was used and the soil was prepared keeping 
a very small height of fall between the spoon and the free 
surface of the sample. For dense samples, the material was 
prepared in fourteen layers of, approximately, 10 mm in 
thickness. Compaction was applied for each layer using a 
stainless steel tamper with diameter about 0.6 times the 
diameter of the sample and a weight of about 9 kN. The 
height of drop of the tamper was approximately equal to 
40 mm and the total number of tips for the fourteen layers was 
equal to approximately 1050 (i.e., on average 75 tips per 
layer). Assuming a diameter of the sample of 70 mm and a 
length of 140 mm, the applied compaction energy was about 
700 kN m/m³. This compaction energy is slightly greater than 
the energy provided in the specifications of ASTM for standard 
compaction (600 kN m/m³) which resulted in high density

![Fig. 2. Grain size distribution curves of materials tested in a Dnevich-type resonant column apparatus (primary dynamic testing program).](Image)
samples. It is noted that the fractions of the study were coarse-grained, and thus, the difference in $e_{\text{max}}$ and $e_{\text{min}}$ is expected to be narrow (Menq, 2003). This is opposite to the trend observed in clays, for which soils small changes in the overconsolidation ratio can produce significant changes in the void ratio. Therefore, small changes in the compaction energy in the study, and thus, small changes in the void ratio at which the samples were prepared could reflect significant changes in the initial relative density, as depicted in Table 2.

For each specimen, prior to the first isotropic consolidation stage in the resonant column at $p' = 25$ kPa, a vacuum of 5 kPa was applied during the setup of the drive mechanism and the surrounding electrical and mechanical parts of the resonant column. After the setup of the apparatus, each sample was subjected to resonant column testing in torsional mode of vibration at increasing steps of $p'$ equal to 25, 50, 100 and 200 kPa. The aim of the primary dynamic testing program was to (a) measure elastic moduli $G_0$ and material damping $D_{s0}$ of the potential fill and backfill materials in the range of very small strains; (b) study the effects of isotropic effective stress $p'$, void ratio $e$ and grain size characteristics on the dynamic constant-state properties of the samples; (c) develop empirical-type equations that can be used for modeling the elastic properties of granular soils, in particular, coarse-sands and gravels of very low sphericity and angular grains. A link between the soil properties at the grain scale and the overall macro-scale response of the samples, and comparisons between the results of this study and those of previously published data are also discussed. Details of the specimens of the primary dynamic testing program, including the initial void ratio, the dry unit weight and the relative density at which the specimens were prepared, as well as the range in shear strain amplitudes at which $G_0$ and $D_{s0}$ were measured (denoted as $\gamma_{\text{LA}}$), are summarized in Table 2. It is noted that for the relatively low pressures adopted in the study, no significant damage to the grains or breakage of the asperities was observed, at least visually. As mentioned in the studies by Senetakis et al. (2013a, 2013b), for crushable (volcanic) sands and gravels, some damage to the particles was observed visually after the resonant column tests. This was not the case for the hard particles included in the primary dynamic testing program of the present study.

The samples were vibrated in torsional mode using a fixed-free resonant column apparatus of the Drnevich type (Drnevich, 1967). This system utilizes four coils which surround the drive mechanism embedded with two magnets and an accelerometer to record the sample response on its top.

<table>
<thead>
<tr>
<th>Laboratory material</th>
<th>$d_{50}$ (mm)</th>
<th>$C_u$</th>
<th>$C_c$</th>
<th>$d_{\max}$ (mm)</th>
<th>$e_{\text{min}}$</th>
<th>$e_{\text{max}}$</th>
<th>Gravel content (%)</th>
<th>USCS Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAMPLE01</td>
<td>1.33</td>
<td>2.13</td>
<td>1.01</td>
<td>2.00</td>
<td>0.502</td>
<td>0.876</td>
<td>0</td>
<td>SP RC-T</td>
</tr>
<tr>
<td>SAMPLE02</td>
<td>1.33</td>
<td>11.8</td>
<td>0.68</td>
<td>9.53</td>
<td>0.271</td>
<td>0.511</td>
<td>20</td>
<td>SP-SW RC-T</td>
</tr>
<tr>
<td>SAMPLE03</td>
<td>2.00</td>
<td>2.50</td>
<td>1.07</td>
<td>4.75</td>
<td>0.467</td>
<td>0.810</td>
<td>0</td>
<td>SP RC-T</td>
</tr>
<tr>
<td>SAMPLE04</td>
<td>2.00</td>
<td>5.40</td>
<td>0.50</td>
<td>9.53</td>
<td>0.370</td>
<td>0.651</td>
<td>21</td>
<td>SP RC-T</td>
</tr>
<tr>
<td>SAMPLE05</td>
<td>2.00</td>
<td>7.30</td>
<td>0.65</td>
<td>12.7</td>
<td>0.302</td>
<td>0.633</td>
<td>25</td>
<td>SP-SW RC-T</td>
</tr>
<tr>
<td>SAMPLE06</td>
<td>3.07</td>
<td>1.53</td>
<td>0.90</td>
<td>4.75</td>
<td>0.572</td>
<td>0.950</td>
<td>0</td>
<td>SP RC-T</td>
</tr>
<tr>
<td>SAMPLE07</td>
<td>3.00</td>
<td>2.45</td>
<td>1.10</td>
<td>6.35</td>
<td>0.465</td>
<td>0.818</td>
<td>15</td>
<td>SP RC-T</td>
</tr>
<tr>
<td>SAMPLE08</td>
<td>3.07</td>
<td>4.24</td>
<td>1.77</td>
<td>9.53</td>
<td>0.362</td>
<td>0.654</td>
<td>20</td>
<td>SP RC-T</td>
</tr>
<tr>
<td>SAMPLE09</td>
<td>2.90</td>
<td>5.95</td>
<td>1.19</td>
<td>9.53</td>
<td>0.450</td>
<td>0.880</td>
<td>30</td>
<td>SP-SW RC-T</td>
</tr>
<tr>
<td>SAMPLE10</td>
<td>3.00</td>
<td>7.85</td>
<td>0.68</td>
<td>9.53</td>
<td>0.424</td>
<td>0.849</td>
<td>40</td>
<td>SP-SW RC-T</td>
</tr>
<tr>
<td>SAMPLE11</td>
<td>3.00</td>
<td>12.5</td>
<td>0.94</td>
<td>9.53</td>
<td>0.330</td>
<td>0.590</td>
<td>40</td>
<td>SP-SW RC-T</td>
</tr>
<tr>
<td>SAMPLE12</td>
<td>5.50</td>
<td>1.17</td>
<td>0.96</td>
<td>6.35</td>
<td>0.688</td>
<td>0.985</td>
<td>100</td>
<td>GP RC-T</td>
</tr>
<tr>
<td>SAMPLE13</td>
<td>6.40</td>
<td>2.70</td>
<td>1.19</td>
<td>12.7</td>
<td>0.453</td>
<td>0.820</td>
<td>75</td>
<td>GW RC-T</td>
</tr>
<tr>
<td>SAMPLE14</td>
<td>7.80</td>
<td>1.22</td>
<td>0.94</td>
<td>9.53</td>
<td>0.593</td>
<td>0.870</td>
<td>100</td>
<td>GP RC-T</td>
</tr>
<tr>
<td>SAMPLE15</td>
<td>10.1</td>
<td>1.03</td>
<td>1.00</td>
<td>12.7</td>
<td>0.590</td>
<td>0.902</td>
<td>100</td>
<td>GP RC-T</td>
</tr>
</tbody>
</table>

(2) Mean grain-size; (3) coefficient of uniformity; (4) coefficient of curvature; (5) maximum grain size; (6) minimum void ratio; (7) maximum void ratio; (8) percentage of coarse soil retained on No. 4 (4.75 mm) sieve; (9) ASTM D2487-00; (10) RC-T = resonant column torsional mode.

Low-amplitude RC tests were performed at increasing steps of $p'$ equal to 25, 50, 100 and 200 kPa. Fig. 3. Optical microscope image of fine-sand portion of granular material of primary dynamic testing program (fraction 0.075–0.180 mm).
A close-up view of the top of the sample with the attached excitation mechanism and the surrounding coils is given in Fig. 4. The experiments were controlled and monitored manually through an electronic system of controllers, oscillator and amplifiers. Changes in sample height during the consolidation stage or during dynamic loading were monitored through a vertically positioned linearly variable differential transformer (LVDT) of repeatability equal to 0.01 mm. This means that the transducer could capture changes in the sample length not less than 0.01 mm. In those tests and for the case of dry specimens, the estimated changes in sample volume were implemented by the records of the vertically positioned LVDT assuming isotropic compression of the sample through Eq. (3), where $\varepsilon_v$ and $\varepsilon_a$ are the volumetric and axial strains, respectively.

$$\varepsilon_v = 3 \times \varepsilon_a$$

For the analysis of the resonant column tests, the ASTM specifications were adopted (ASTM, 1992), whilst for the material damping derivations, the steady-state vibration method was used (Senetakis et al., 2015).

### 2.2 Secondary dynamic testing program: analysis of literature test data

Apart from the samples of the primary dynamic testing program (Tables 1 and 2), additional granular soils, for which
wave propagation velocities and moduli have been presented in previous published works by the authors, were included in the study. These soils are summarized in Table 3 and referred to as samples of the secondary dynamic testing program. Quartz sand SP-2 was studied by Madhusudhan (2011); this soil was subjected to torsional and flexural modes of vibration in a modified Stokoe-type resonant column apparatus (Cascante et al., 1998). Quartz sand SP-4 was studied by Kumar and Madhusudhan (2010); this soil was subjected to bender/extender element tests; and thus, shear and Young’s moduli were measured for both SP-2 and SP-4. Details of the bender/extender element method and the adopted interpretations may be found elsewhere (e.g., Kumar and Madhusudhan, 2010, 2012). The soils with code names LWCID6 (pumice gravel) and V3 (rhyolitic crushed rock) in Table 3 were studied by Senetakis et al. (2012, 2013a, 2013b); these soils were subjected to a torsional mode of vibration under isotropic resonant column testing.

All the materials included in Table 3 were prepared at three to five initial void ratios; and thus, these samples were included in this study in order to (a) enrich the interpretations of this research work with respect to the effect of the preparation method and the initial fabric on the elastic modulus parameters, (b) highlight the importance of the particle contact response in the constant-state behavior of particulate media by comparing the effect of $p’$ in the changes in void ratio in samples of variable types (i.e., hard or weak-crushable grains) and (c) provide, based on the experiments, useful equations that can be utilized in numerical simulations of particulate media (i.e., discrete element simulations) associated with changes in the void ratio of isotropically consolidated samples of variable types for a given change in $p’$. It is noted that sands SP-2 and SP-4 are quartz of strong-massive particles, whereas materials LWCID6 and V3 are volcanic soils of weaker particles of intra-particle voids. Therefore, the interpretations for those soils comprise the key for a link between properties at the grain scale and macro-scale observed responses associated with constant-state properties.

3. Experimental results and discussion

3.1 Small-strain shear modulus: formulation of empirical-type equation, typical results and comparisons with literature models

Typical plots of small-strain shear modulus $G_0$ against $p’$ are given in Fig. 5(a) and (b) for uniform and well-graded samples, respectively. Fitting curves of the general formula of Eq. (1) are also depicted in these figures along with the computed values for the power $n_G$.

For the uniform soil (SAMPLE06), this power corresponded to values in a range of $n_G = 0.45$–0.55, which are typical values for reconstituted uniform granular soils reported in the literature (e.g., Hardin and Richart, 1963; Hardin, 1978; Kokusho, 1980; Santamarina et al., 2001). For the well-graded soil (SAMPLE04), $n_G$ had higher values, which was the general trend for the well-graded soils of the study in comparison to uniform samples.

In Fig. 6, the measured $G_0$ values of the study were compared with empirical-type models proposed in the literature. In particular, the formulae proposed by Hardin and Richart (1963) in Fig. 6(a), Menq (2003) in Fig. 6(b) and Wichtmann and Triantafyllidis (2009) were used for this purpose. It was noticed that these formulae underestimated the measured shear moduli and this may be attributed in part to differences in the particle shape descriptors between the soils of the present study and the soils used for the development of the empirical-type models in previous studies. More pronounced scatter was noticed at higher levels of $p’$ which mirrors the possible scatter of the $n_G$ values. On the other hand, the underestimation of the shear moduli mirrors the differences in $A_G$ values between the empirical models and the data of the study. In previous works, for example, by Menq (2003) or Wichtmann and Triantafyllidis (2009), the grading characteristics were incorporated into the development of the $G_0$ formulae, but it is possible that these formulae may more efficiently cover soils of sub-angular to rounded particles.

Table 3

<table>
<thead>
<tr>
<th>Laboratory material</th>
<th>$d_{50}$ (mm)</th>
<th>$C_u$</th>
<th>$C_c$</th>
<th>$d_{max}$ (mm)</th>
<th>Gravel content (%)</th>
<th>USCS</th>
<th>Type</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
<td>(9)</td>
</tr>
<tr>
<td>SP2</td>
<td>0.31</td>
<td>2.00</td>
<td>1.08</td>
<td>0.42</td>
<td>0</td>
<td>SP</td>
<td>Quartz</td>
<td>RC-T/F</td>
</tr>
<tr>
<td>SP4</td>
<td>2.50</td>
<td>1.32</td>
<td>0.88</td>
<td>4.75</td>
<td>0</td>
<td>SP</td>
<td>Quartz</td>
<td>BE/EE</td>
</tr>
<tr>
<td>V3</td>
<td>0.55</td>
<td>4.18</td>
<td>0.75</td>
<td>4.75</td>
<td>0</td>
<td>SP</td>
<td>Rhyolite</td>
<td>RC-T</td>
</tr>
<tr>
<td>LWCID6</td>
<td>5.60</td>
<td>1.20</td>
<td>0.97</td>
<td>9.53</td>
<td>96</td>
<td>GP</td>
<td>Pumice</td>
<td>RC-T</td>
</tr>
</tbody>
</table>

(2) Mean grain-size; (3) coefficient of uniformity; (4) coefficient of curvature; (5) maximum grain size;
(6) percentage of coarse soil retained on No.4 (4.75 mm) sieve; (7) ASTM D2487-00
(8) Type of soil;
(9) RC-T/F=resonant column torsional and flexural modes, RC-T=resonant column torsional mode BE/EE=bender and extender elements.
(a) Madhusudhan (2011).
(b) Kumar and Madhusudhan (2010).
(c) Senetakis et al. (2012, 2013a).
(d) Senetakis et al. (2013b).
whilst in the present study, the grains were of high angularity. Cho et al. (2006) highlighted the importance of particle shape in both $A_G$ and $n_G$ parameters. Thus, a modification of the constants of those empirical formulae is necessary for a better fit to granular soils of high angularity.

3.2 Effect of isotropic pressure on changes in void ratio

In addition to the effect of $C_u$ on the small-strain shear modulus parameters, the RC test results in Fig. 5 and for a given sample, showed higher values for $n_G$ for a looser specimen than a denser one, and this was a systematic trend observed in the study. In order to further examine the effect of the sample preparation method on $n_G$, the changes in void ratio $\Delta e$, of the samples, due to the change in $p'$, were estimated by adopting the formulae from the theory of elasticity expressed through Eqs. (4) and (5) (Richart et al., 1970). These formulae may provide an alternative estimation of the changes in sample void ratio due to an increase in $p'$. Typically, changes in void ratio are implemented by assuming isotropic compression.

$$\Delta e = \frac{p' \times (1 + e_0)}{K}$$  \hspace{1cm} (4)

$$K = \frac{E_0}{3} G_0$$  \hspace{1cm} (5)

In Eqs. (4) and (5), $K$ is the bulk modulus, $e_0$ is the void ratio at which the samples were prepared, $G_0$ is the measured small-strain shear modulus and $E_0$ is the small-strain Young's
modulus. For the computation of \( E_0 \), Eq. (6) was used by
assuming a value for Poisson’s ratio, equal to 0.25, which
might be representative for granular soils (Menq, 2003).
\[
E_0 = 2 \times (\nu + 1) \times G_0 
\]

The change in void ratio \( \text{de} \), for a given increase in \( p' \) for
SAMPLE04 and SAMPLE06, is given in Fig. 7. Through a
regression analysis, the fitting curves of the general formula of
Eq. (7) were also plotted in the figure. Parameter \( n_e \) in Eq. (7)
expresses the slope of the \( \text{de} - p' \) relationship, whilst \( \beta \) is a
constant, and thus, \( n_e \) mirrors the alteration in constant-state
properties of granular soils for a given change in \( p' \).
\[
de = n_e \times p' + \beta 
\]

From the results in Fig. 7 it was demonstrated that the exponent \( n_e \)
decreased with a decrease in void ratio or, alternatively, with an increase in relative density. These
observations demonstrated that the power \( n_e \), and thus, the
\( G_0 - p' \) relationship, is possibly affected by the preparation
method, and that the more pronounced increase in \( G_0 \) with an
increase in \( p' \), for a looser sample, is attributed to the more
pronounced changes in void ratio when the sample has a lower
relative density. This observed trend might be explained by the
fundamental behavior of soils reported by Jovicic and Coop
(1997). In particular, Jovicic and Coop found that the samples
they tested, prepared with different initial void ratios, tended to
have a unique line in the log \( G_0 - \log p' \) plot and this observed trend
was more pronounced at relatively high isotropic pressures, in
general beyond 1.0 MPa. In the present study and in most soil
dynamics laboratory research works, the pressures under
consideration are in the range of 0.05–0.5 MPa for most
practical purposes. Therefore, the slight increase of \( n_e \) for a
looser packing, may be related to the trend of the initial moduli
at variable relative densities and for a given soil to converge at
higher pressures to a unique line.

It is noted that, in this study, the investigation of the effect of the
preparation method on the shear modulus parameters was
limited because only two specimens were tested in the resonant
column for each sample of the primary dynamic testing
program, namely, a relatively dense to very dense sample
prepared with compaction in layers and a loose to very loose
sample prepared with the hand-spooning method. For the sake
of completeness and in order to further study the aforementioned
interpretations, the authors present herein corresponding
results by Madhusudhan (2011) and Kumar and Madhusudhan
(2010) who thoroughly investigated the effect of the sample
preparation method on the shear modulus parameters of a fine-
grained uniform quartz sand denoted as SP2 and a coarse-
grained uniform sand denoted as SP4 (Table 3). The \( \text{de} - p' \)
relationships from those two studies are presented in Figs. 8
(a) and 9(a) along with estimated fitting curves using Eq. (7).
The \( n_e \)-RD relationships are plotted in Figs. 8(b) and 9(b).
These results, in agreement with the findings of the primary
dynamic testing program, demonstrated the importance of the
sample preparation method in the \( G_0 - p' \) relationship with a
decrease of the power \( n_e \) for denser samples.

3.3 Effect of particle type and particle contact response on de–p’ relationship

The importance of soil type and the nature of the particle
contact response on the \( \text{de} - p' \) relationship was examined by a
comparison between the results of Figs. 7–9 with the results
shown in Fig. 10. In the latter figure, the corresponding \( \text{de} - p' \)
and \( n_e \)-RD relationships for sands and gravels of volcanic
origin with intra-particle voids are presented. The pumice soil,
tested by Senetakis et al. (2013b), named LWC1D6 in that
study, and the crushed rhyolithic rock, tested by Senetakis
et al. (2012, 2013a), named V3 in their work, were subjected to
torsional mode of vibration; and thus, only the \( G_0-p' \) relationships
were available in that study. The authors back-calculated
Young’s modulus from Eq. (6) assuming a Poisson’s ratio of
0.25 for the volcanic soils. Using Eqs. (4)–(7) and through a
regression analysis, the \( \text{de} - p' \) and \( n_e \)-RD relationships were
thereafter determined. It was revealed that the values for \( n_e \)
of the volcanic soils were much higher than the \( n_e \) values of the
soils of this study (Fig. 7) and the studies by Madhusudhan
(2011) (Fig. 8) and Kumar and Madhusudhan (2010) (Fig. 9)
with additional substantially higher (de) values for the volcanic
soils, in particular for the pumice gravel. For example, for the
pumice gravel in Fig. 10, the \( n_e \) values ranged between 0.13...
and 0.25, whereas for the soils of stronger particles tested by Madhusudhan (2011), Kumar and Madhusudhan (2010) or the samples of this study, the $n_e$ values ranged between 0.04 and 0.09.

Consequently, along with the effect of the sample preparation method on the $G_0-p'$ relationship, these interpretations highlighted the importance of soil type and particle contact response in the overall behavior of particulate media derived from dynamic element tests. These observations may be attributed, in part, to the plastic nature of the particle contact response in both the normal and tangential directions at the particle contacts. It is assumed, considering the recently acquired micromechanical test data by Cole et al. (2010) and Senetakis et al. (2013c, 2013d), that the response at the contacts of volcanic particles is more brittle with more pronounced damage to the grain surface which, in turn, mirrors the more pronounced fabric changes due to a change in $p'$ in comparison to the assemblies of stronger particles. These observations may need to be considered when modeling the behavior of soils, for example, isotropically consolidated samples in discrete element analyses. Thus, the $de-p'$ relationships in Figs. 7–10 may provide a link in this direction. It is noted that the more pronounced crushing for the volcanic grains is attributed primarily to the effect of the increase in isotropic pressure and not to the damage due to the sample preparation, which was verified by trial tests conducted when preparing the dense samples and by visually examining the possible damage to the grains due to compaction.

3.4 Development of empirical-type equations based on the primary RC testing program

In Fig. 11, the small-strain shear moduli are presented in log plots against $p'$ for all samples of the primary dynamic testing program. For further analysis and interpretations for $G_0$, Eq. (8) was used, in which $F(e)$ is a void ratio function and $p'$ is normalized with respect to the atmospheric pressure $p_a$. It is noted that parameter $A_G^*$ in Eq. (8) is different in magnitude than parameter $A_G$ in Eq. (1) because the effect of the void ratio is considered in the $F(e)$ function herein and $p'$ is expressed by means of the normalized value $p'/p_a$. In Fig. 11, the samples have been separated into four groups; Fig. 11(a) refers to sands of $D_{50} \approx 1$ mm (Group A), Fig. 11(b) refers to sands of $D_{50} \approx 2$ mm (Group B), Fig. 11(c) refers to sands of $D_{50} \approx 3$ mm (Group C) and Fig. 11(d) refers to uniform, well-graded gravels (Group D).
A. C. Jamiolkowski et al., 1991) was adopted. Therefore, Eq. (1) is rewritten as follows:

\[ G_0 = A_G^* \times F(e) \times \left( \frac{p'}{p_a} \right)^{n_G} \]  

(8)

\[ F(e) = \frac{1}{e^{x_e}} \]  

(9)

For \( F(e) \), the general formula presented in Eq. (9) was used in which a value for the power \( x_e \) equal to 1.3 (after Jamiołkowski et al., 1991) was adopted. Therefore, Eq. (1) is rewritten as follows:

\[ G_0 = A_G^* \times \frac{1}{e^{1.3}} \times \left( \frac{p'}{p_a} \right)^{n_G} \]  

(10)

Using the experimentally derived \( G_0 \) values, the constants \( A_G^* \) and \( n_G \) were computed by fitting the \( G_0-p' \) relationship with the power-law type function. These values are summarized in Table 2. By plotting \( A_G^* \) and \( n_G \) against \( C_u \), as shown in Fig. 12, fitting curves were estimated that correlated the small-strain shear modulus constants with the coefficient of uniformity. The proposed formulae are given in Eqs. (11) and (12).

\[ A_G^* = -3.36 \times C_u + 81.8 \]  

(11)

\[ n_G = 0.485 \times C_u^{0.13} \]  

(12)

For the shear moduli, the measured values are plotted against the predicted values in Fig. 13 using Eqs. (11) and (12). The scatter between the measured and the estimated values varied for most samples within a range of \( \pm 20\% \) which is satisfactory for practical purposes. These formulae may be more applicable for coarse-grained materials of very low sphericity and high angularity grains as well as for the normalization of \( G/G_0 \) against shear strain curves, for example, by means of a hyperbolic type of model (Menq, 2003, Senetakis et al., 2013a, 2013b), since \( G_0 \) defines the plateau of the normalized curves. The scatter shown in Fig. 13 is significantly reduced in comparison to the results in Fig. 6, which is related, in part, to the variability in particle shape descriptors used in different research works to develop empirical-type models.

It is noted that, in Eq. (12), the relative densities of the samples were not incorporated into the \( n_G \) power due to the limited number of specimens prepared from each sample; and thus, the equation is applicable for relatively low engineering working loads. It is also noted that Eq. (12) would predict a value for \( n_G \) equal to approximately 0.49, for \( C_u=1 \), which is very close to the typical value of 0.50 reported in the literature for the contact response of real-soil particles which, in nature, is visco-plastic to brittle. On the other hand, for \( C_u=5 \), Eq. (12) would predict a value for \( n_G \) equal to 0.60. Similarly, previous research works have indicated an increase in \( n_G \) with an increase in \( C_u \). In different studies, however, the different formulae and magnitudes of \( n_G \) mirror, in part, the differences in particle shape and overall morphology (for example, the research works by Menq, 2003, Wichtmann and Triantafyllidis, 2009, and the results of the present study). In particular, the empirical-type equations proposed by Menq (2003) and Wichtmann and Triantafyllidis (2009) for the \( n_G \) prediction as a function of \( C_u \) are plotted for comparison in Fig. 12(b).

On the other hand, Eq. (11) would predict a decrease in \( A_G^* \) with increasing \( C_u \), which implies that for a given void ratio and \( p' \), the constant-state stiffness decreases in magnitude for well-graded soils. This observation may be analytically explained through Eq. (13), in which, \( f_n \) is the average contact force for an isotropic particulate medium of spherical particles of a given size, \( e \) is the void ratio, \( C_u \) is the average coordination number within the granular assembly, \( p \) is the isotropic pressure and \( r \) is the radius of the particles in contact (Rothenburg and Bathurst, 1989, after Cascante and Santamarina, 1996). This formula provides an indication and a strong link to the findings of this work on that, for a given \( e \) and pressure, the normal contact force, as an average value within a granular material, decreases with an increase in the coordination number and the latter is a function of the coefficient of uniformity.

\[ f_n = \frac{4 \times \pi \times (1 + e) \times r^2 \times p}{C_n} \]  

(13)
3.5 Small-strain material damping: formulation and development of empirical equation

The measured small-strain material damping values of all samples of the primary dynamic testing program are plotted in Fig. 14 against the normalized pressure in log plots. A slight decrease in $D_{s0}$ with an increase in pressure, which is consistent with previous research works (e.g., Cascante and Santamarina, 1996; Menq, 2003; Senetakis et al., 2012), was observed. For $p' = 25$ kPa, material damping ranged between 0.80% and 1.10% for most samples, whilst for $p' = 200$ kPa, $D_{s0}$ ranged between 0.45% and 0.75% in the majority of the experiments. No clear trend was observed for the effect of the initial void ratio on $D_{s0}$; for example, for SAMPLE07 or SAMPLE15, $D_{s0}$ values decreased for the denser specimens in comparison to the looser samples, but the opposite trend was observed for SAMPLE06 or SAMPLE14. It is noted that, in the study, the changes in sample volume, and thus, in void ratio, were based on the assumption of isotropic compression. In a previous study by Senetakis (2011), saturated samples at variable initial densities were tested and accurate measurements of changes in sample volume were incorporated. Senetakis (2011) did not notice a systematic effect of the void ratio on small-strain damping, which is in agreement with a previous work on granular soils by Menq (2003).

In order to further analyze the data, the following general formula for material damping was adopted which is given as a function of the normalized pressure.

$$D_{s0} = A_D^* \times \left( \frac{p'}{p_a} \right)^{n_D}$$  \hspace{1cm} (14)

In Eq. (14), $A_D^*$ and $n_D$ are material constants. For those constants, a similar procedure to the one followed for the determination of the shear modulus constants was adopted. For each sample, $D_{s0}$ was plotted against $p'/p_a$ and a power law equation was determined expressed through the constant $A_D^*$ and the power $n_D$. The analyses for those constants are summarized in Table 2, while in Fig. 15, $A_D^*$ and $n_D$ are plotted against the coefficient of uniformity. No clear trend was observed for the effect of the initial void ratio or the grading characteristics of the samples on $D_{s0}$. The data indicated an average value and a standard deviation for $A_D^*$ and $n_D$ equal to 0.71 ± 0.12% and −0.17 ± 0.10, respectively. The material damping constants were more scattered than the corresponding stiffness constants; this observation may be...
attributed to the overall complex mechanisms of energy dissipation in particulate media at very small deformations. In their analyses, Senetakis et al. (2012), based on resonant column test data on fine to coarse grained sands of variable types including shape, mineralogy and morphology, similarly reported average values for the constant $A_D$ between 0.62% and 0.52% for quartz and volcanic sands, respectively, whilst the $n_G$ values were affected by particle shape. In Fig. 16, the measured against the estimated damping ratios using Eq. (14) are plotted. The use of Eq. (14) with $A_D = 0.71\%$ and $n_D = -0.17$ did not show any systematic overestimation or underestimation of $D_{so}$ values over the measured damping ratios. For most data points, the scatter between the measured and estimated values varied within a range of $\pm 30\%$. This scatter is satisfactory considering the uncertainties in measuring material damping in the laboratory.

3.6 Practical implications of proposed formulae and future improvements

Computer codes for seismic response analysis studies use $G_0$ (or shear wave velocity $V_s$) and non-linear curves, i.e., modulus degradation and increase in damping against shear strain, as input. In addition, $G_0$ is important in the normalization of the $G/G_0$-strain curves which are very popular in engineering practice particularly when equivalent linear analysis codes are used for seismic response studies and in medium-strain geotechnical problems. $G_0$ is the "start point" of a normalized curve and it has been recognized for its importance in the prediction of deformations for both static and dynamic loading problems (e.g., Jovicic and Coop, 1997). Consequently, the importance of small-strain stiffness is associated not only with studies that include seismic loading, but also with conventional foundation engineering, tunneling design, infrastructures such as retaining walls and other facilities with fill–backfill material. Damping in the range of small-strains can also be very important in particular in small-strain problems which may be of interest in machine foundation vibration analyses and soil-structure-interaction analyses of dynamically loaded systems. Small-strain damping is also important for the geophysical characterization of sediments (Cascante et al., 1998). This means that the dynamic properties examined in the study are of immediate interest in engineering practice, including deformation prediction and dynamic problems, and the produced formulae of the paper can be used directly for predictions of ground deformation as well as for geophysical characterizations of sediments.

The aim of the study was to propose formulae for small-strain stiffness and damping prediction that are applicable to soils of similar characteristics in terms of particle size, distribution, specific gravity and most importantly particle shape descriptors, i.e., for soils of low sphericity and roundness. This type of soil, such as crushed rock, is very common in geotechnical projects.

It is noted that in the present study and, in particular, for stiffness derivations, a void ratio function proposed in the literature was incorporated, i.e., $F(e) = e^{-x}$ where the power $x$ was equal to 1.3. In previous studies, for example by Menq (2003) or Senetakis et al. (2012), the experimental data were adjusted in order to compute a best-fit $x$ power, but in the present work a constant value was adopted, without adjustment based on the specific experiments of the study. This may add some additional scatter to the predicted stiffness values. The decision for an appropriate void ratio function $F(e)$ to be used in the analysis affects the magnitude of the constant $A_G$, but it
is expected not to have a qualitatively important effect on general trends, for example, the effect of $C_u$ on the power $n$ or the constant $A_G$, whilst the produced formulae demonstrated an improvement between the estimated and the measured values. It was out of the scope of this paper to investigate in further detail the uncertainties in estimating stiffness at small strains, which may include, for example, the effect of different void ratio functions to be used in the small-strain stiffness formula. The main contributions of this work were (1) to emphasize some differences with the literature data, for example, between different formulae for stiffness prediction; (2) to highlight the important effect of particle shape which significantly affects the differences in the proposed formulae in the literature; (3) to improve the general formula for $G_0$ prediction for the
particular type of soil which was mirrored through the improved comparison between the measured and the estimated values; and (4) to provide micro-mechanical interpretations and the effect of the particle contact response including soils of variable types.

4. Conclusions

The study presented a synthesis of torsional resonant column tests on a potential fill–backfill material with focus on elastic moduli $G_0$ and material damping $D_{00}$. Additional previously published tests that adopted dynamic test methods were re-analyzed. The following main conclusions may be drawn:

1. The $G_0$–$p'$ relationship followed a power law with observed effects of the preparation method and the coefficient of uniformity on the power $n_{00}$. These effects were attributed, in part, to the fundamental behavior of soils of convergent moduli with respect to the effect of the preparation method and to the effect of the coordination number on the magnitude and the distribution to the particle contact forces within a granular assembly, with respect to the coefficient of uniformity.

2. Using formulae from the theory of elasticity, along with previously published data from dynamic element tests, the importance of particle type and particle contact response was examined in terms of the changes in void ratio for a given change in $p'$. The more brittle in nature particle-contact response of weaker particles, such as volcanic soils, led to more pronounced changes in void ratio, and thus, changes in fabric with an increase in $p'$, which in turn affected the $G_0$–$p'$ relationship.

3. Material damping constants were more scattered and the effect of grading or the preparation method on $D_{00}$ was not clear. For the range of isotropic effective stresses in this study, small-strain material damping was generally found to be less than unity in magnitude, with a slight decrease with an increase in $p'$.

4. The developed equations of this research work may be directly used to model the behavior of potential fill–backfill materials which may find many applications in geotechnical engineering practice. These formulae may be more applicable to granular soils of very high angularity and very low sphericity. However, it is believed that further laboratory and theoretical research, by means of numerical simulations of granular assemblies, is necessary in order to examine some observed trends of this work. For example, the effect of the sample preparation method and its link to the magnitude and the distribution of particle contact forces, attenuation at the contacts of the soil particles and its link to the macro-scale observed material damping, or, the effects of particle shape and morphology in the energy dissipation mechanisms and stiffness of particulate media, need to be investigated.

Acknowledgments

The authors would like to thank the anonymous reviewers for their constructive comments and their detailed suggestions which helped us to improve the quality of the paper.

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