

## Technical Note

# A comparison of material damping measurements in resonant column using the steady-state and free-vibration decay methods



Kostas Senetakis<sup>a,\*</sup>, Anastasios Anastasiadis<sup>b</sup>, Kyriazis Pitilakis<sup>b</sup>

<sup>a</sup> School of Civil and Environmental Engineering, University of New South Wales, Sydney, Australia

<sup>b</sup> Department of Civil Engineering, Aristotle University of Thessaloniki, Thessaloniki, Greece

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## ABSTRACT

The study reports results associated with the small to medium strain range material damping of quartz sand. The experiments were conducted in a fixed-free type resonant column and the samples were subjected to torsional mode of vibration at their first natural frequency. A comparison between the derived damping values using two different methods is presented: the steady-state vibration method (SSV) and the free-vibration decay method (FVD). Within the scatter of the data the two different methods in measuring material damping provided comparable results with a scatter, in most cases, of less than  $\pm 15\%$  for the working strain range of the resonant column method. The damping values derived from the FVD and SSV methods were also compared with proposed models in the literature by means of stiffness degradation–damping increase correlation.

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## 1. Introduction

Dynamic properties of soils are commonly expressed in terms of the secant shear modulus, denoted as  $G_s$ , and material damping in shear, denoted as  $D_s$ .  $G_s$  and  $D_s$  are essential input parameters in computer codes that use linear iterative approaches, such as, for example, the codes SHAKE [6] and QUAD4M [3]. The resonant column (RC) method has been established as a standard laboratory non-destructive procedure for the evaluation of the elastic modulus  $G_{max}$  as well as  $G_s$  and  $D_s$  as a function of shear strain amplitude [2] with working strain levels within a range of about  $10^{-4}\%$  to about  $10^{-2}\%$ .

In resonant column testing there are commonly two different approaches in measuring material damping: (a) during a steady-state vibration, i.e. during the sample is vibrated at its first mode, denoted as the SSV method and (b) during free-vibration decay, i.e. there is cut off the constant vibration of the sample at resonance and the specimen is allowed to free vibration mode whilst the decayed strain amplitude during free vibration is measured, denoted as the FVD method. Both methods are described in ASTM specifications [2]. Recently, damping derivations from the resonant column method in torsional mode of vibration were re-examined introducing micro-mechanical considerations [5].

In the literature comparisons between the two different methods, i.e. SSV against FVD, are relatively limited, while Stokoe et al. [11] or Menq [4] have suggested to use the SSV method when

small strain measurements are conducted and the FVD method when medium strain measurements are carried out. The ASTM specifications do not make it clear if there is a preferable method depending on the strain level. In this direction, this brief note reports resonant column test results performed on quartz sand with a comparison between material damping values derived from both the SSV and FVD methods. It is noted that in previous research works by the authors (e.g. [7,9,10]) a large database of material damping derivations was presented on variable types of sands based on the SSV method. This paper focuses particularly to some comparative results between the two different approaches in evaluating material damping in the range of working strain levels of the resonant column.

## 2. Brief description of materials and methods

### 2.1. Materials and laboratory equipment

Two fractions from a fluvial sand of dominantly quartz particles were used in the study. “Sand1” had a mean grain size ( $d_{50}$ ) equal to 2.76 mm and a coefficient of uniformity ( $C_u$ ) of 2.76. For “Sand2”,  $d_{50} = 1.33$  mm and  $C_u = 2.13$ . Four dense samples from these two sands were constructed in a resonant column apparatus and tested in torsional mode of vibration in both dry and saturated states at variable levels of isotropic effective stress ( $p'$ ). The resonant column (RC) used is of Drnevich type and follows the fixed-free configuration. Details of the preparation of dry and saturated samples and technical features of

\* Corresponding author.

E-mail address: [k.senetakis@unsw.edu.au](mailto:k.senetakis@unsw.edu.au) (K. Senetakis).

the RC apparatus have been thoroughly described in Anastasiadis et al. [1] and Senetakis et al. [7–10].

2.2. Formulae for material damping derivations

In the resonant column, shear modulus ( $G_s$ ) is computed from the known resonant frequency ( $f$ ) and the mass density of the sample ( $\rho$ ) using the following formula:

$$G_s = \rho \times \left[ \frac{f}{FT} \right]^2 \times [2 \times \pi \times f]^2 \tag{1}$$

FT is computed as a function of the polar moment of inertia of the cylindrical sample ( $J$ ) and the attached mass on top of the sample ( $J_a$ ) [2]. The first method in measuring material damping is during a steady-state vibration (SSV), i.e. during excitation of the sample at first mode of vibration while a steady-state voltage is introduced into the system. In this case,  $D_s$  is computed as follows:

$$D_s = \frac{1}{A \times MMF} \times 100\% \tag{2}$$

where  $A$  is a calibration factor and MMF is the magnification factor which is given as a function of the voltage introduced into the coils of the resonant column, the displacement on top of the sample which is derived from the recorded acceleration, and factors derived from calibration exercise [2].

Cutting off the introduced voltage into the system allows the sample to free-vibration decay mode. In this case, material damping is measured from the logarithmic decrement, commonly of two successive cycles during free-vibration:

$$\delta = \ln \left[ \frac{\lambda_1}{\lambda_2} \right] \tag{3}$$

where  $\lambda_1$  and  $\lambda_2$  are the amplitudes of two successive cycles during free-vibration. In this case,  $D_s$  is determined from the following equation [11]:

$$D_s = \sqrt{\frac{\delta^2}{\delta^2 + 4 \times \pi^2}} \times 100\% \tag{4}$$

Material damping derived from Eq. (4) corresponds to the free-vibration decay (FVD) method. A typical plot of strain amplitude against time and introduced voltage into the system against time in which the cut off point is clearly shown is given in Fig. 1. In this figure,  $N$  denotes the number of successive cycle after the free-vibration of the sample is initiated. Stokoe et al. [11] have suggested to use three successive cycles when the FVD method is used for damping derivation, perhaps, of its possible better fit and minimization of ambient noise. In the study, the standard approach of two successive cycles was implemented.

3. Results

$G_{max}$  and ( $G_s/G_{max}$ ) against shear strain ( $\gamma$ ) of the samples of the study have been discussed elsewhere (e.g. [1,7,9]) and thus it was out of the scope of this brief note to present shear modulus derivations. Typical results of material damping using the SSV and FVD methods for a wide range of shear strain amplitudes are given in Fig. 2. These results corresponded to a dry sample of Sand1 tested at  $p' = 200$  kPa. The results indicated a satisfactory comparison between the two methods of material damping computation for both the range of relatively small and small to medium strain ranges.

A comparison between small-strain material damping ( $\gamma < 10^{-3}\%$ ) derived from the SSV and FVD methods for all samples is given in Fig. 3. For sands and for typical isotropic effective stresses from 25 to 400 kPa,  $D_s$  ranges, in general, from about 1.3%

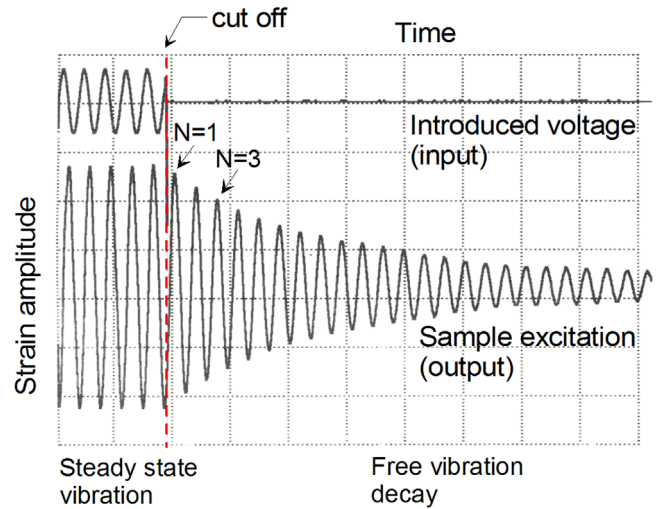


Fig. 1. Typical free-vibration decay of sand during resonant column experiment.

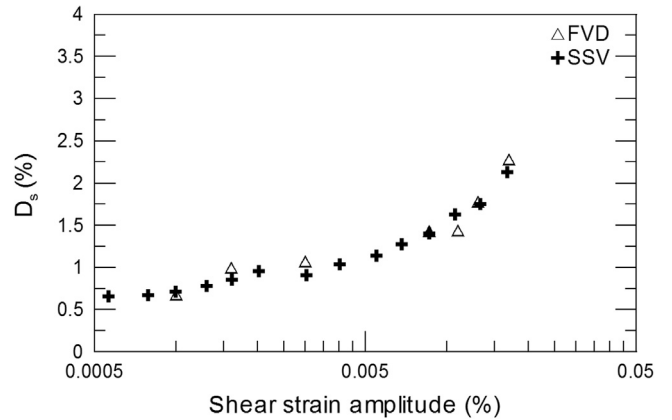


Fig. 2. Material damping against shear strain amplitude: comparison between the steady-state vibration (SSV) and the free-vibration decay (FVD) methods.

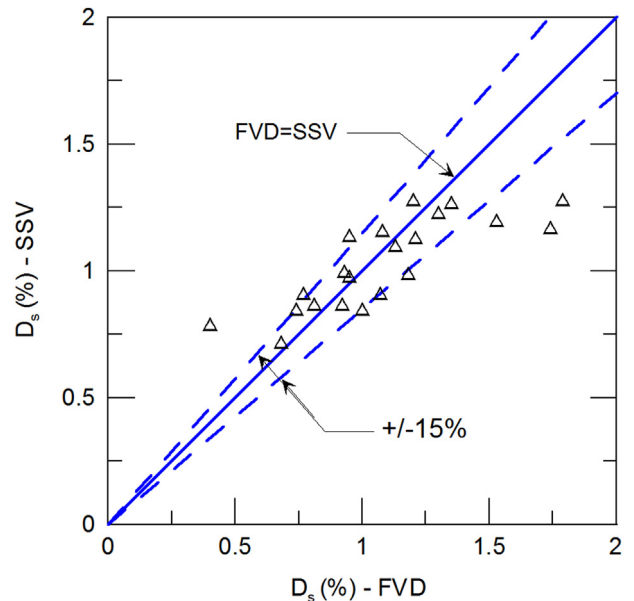


Fig. 3. Material damping values derived at small-strains: comparison between the steady-state vibration (SSV) and the free-vibration decay (FVD) methods.

to 0.4%, with decreasing values as  $p'$  increases and with slightly higher values for saturated than dry samples [4,1,7]. It is seen in the figure that within the scatter of the data and with the exemption of some data points, the steady-state and free-vibration methods provided comparative results with a scatter of  $\pm 15\%$  for most data points. Considering the uncertainties in measuring material damping at small strains, this scatter may be assumed quite satisfactory. In addition, there was not observed a systematic trend of under- or over-estimation of  $D_s$  values when the FVD method is used over the SSV method.

The total number of data points from all samples and from small to medium strains are given in Fig. 4 with a comparison between the derived material damping from the SSV and FVD methods. The scatter in most cases was within a range of  $\pm 15\%$ . As in the case of Fig. 3, there was not observed a systematic trend of under- or over-estimation of  $D_s$  values when one method was used over the other one. Appropriate use of amplifiers which can provide minimum effect of ambient noise and appropriate calibrations of the RC apparatus can provide reliable material damping measurements. In authors experience, a resonant column should be validated for damping derivations based, primarily, on dry sands and gravels as well as saturated granular soils. This is because sandy samples provide a better interaction-friction between top cap and top of sample, whereas in clays there may be two factors that can affect damping derivations: (a) non-satisfactory friction between sample and top cap and (b) in particular for soft clays accurate measurements at very small deformations may be difficult to be obtained without significant effects of ambient noise.

Finally, in Fig. 5 a comparison is made between the damping values derived from the FVD and SSV methods and literature models for dry sands [7,9] and non-plastic natural soils [12] by means of  $D_s$  against  $G_s/G_{max}$  correlation. The theoretical models plotted corresponded to  $p' = 100$  kPa which was an average isotropic stress for the samples of the study, and for this stress level small-strain damping is equal to 0.62% based on Senetakis et al. [7] and equal to 0.82% based on Zhang et al. [12] models. Within the scatter of the data there was observed a satisfactory comparison

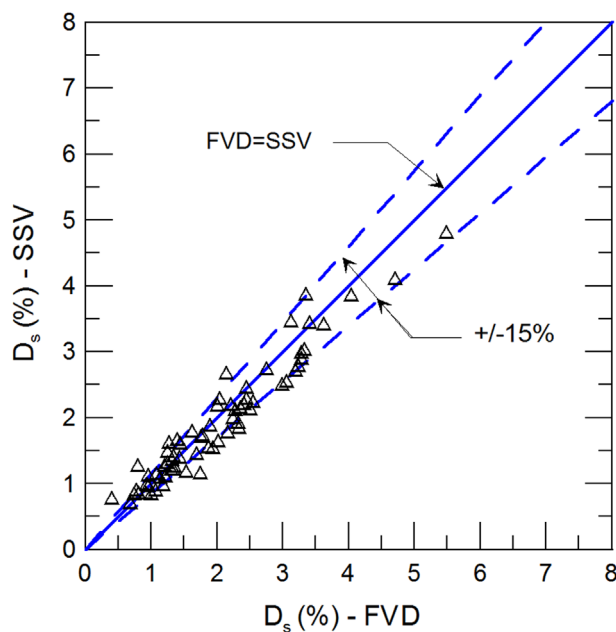


Fig. 4. Material damping values derived at small to medium strains: comparison between the steady-state vibration (SSV) and the free-vibration decay (FVD) methods.

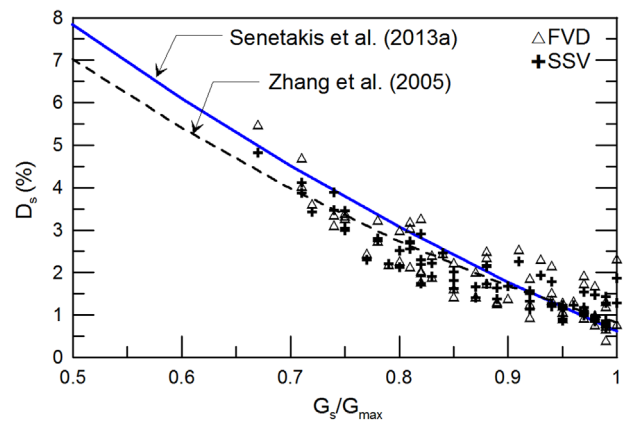


Fig. 5. Material damping against normalized shear modulus: comparison between theoretical models proposed in the literature and values derived in the study using the free-vibration decay and the steady-state methods.

between measured  $D_s$  values using the FVD and SSV methods and the models proposed in the literature.

#### 4. Conclusions

The study reported briefly results derived from torsional resonant column tests on dry and saturated sands. Particular focus of this brief note was the comparison of material damping ( $D_s$ ) values derived from two different methods: (a) the steady-state vibration (SSV) method and the free-vibration decay (FVD) method. In the study, the ASTM specifications were used for damping derivations. For both small-strain and small to medium strain ranges there was observed a satisfactory comparison between the damping values derived from the two methods. There was not observed a systematic over- or under-estimation of the observed values when one method was used over the other one. Within the scatter of the data, the discrepancy of the data when the SSV and the FVD methods were used was within a range of  $\pm 15\%$  for most data points.

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