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BRIEF TECHNICAL NOTE

# Micro-mechanical Experimental Investigation of Grain-to-Grain Sliding Stiffness of Quartz Minerals

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**Abstract** The study presents micro-mechanical tests at the contacts of quartz soil grains exploring the inter-particle sliding (tangential) stiffness through tests of a force-controlled type. The experiments are conducted in a custom-built micromechanical apparatus which is capable to study the particle contact response at relatively low confining forces. This makes its configuration more appropriate for soil mechanics purposes. The experimental data indicated a dependency of particle-contact sliding stiffness ($K_{T0}$) on the level of normal force ($F_N$) with almost a linear increase of $K_{T0}$ with $F_N$ for the range of normal forces of the study. The results of the research work could be incorporated directly as input in numerical simulations that use the discrete element method for a more realistic representation of the behavior at soil grain contacts.

**Keywords** Micro-mechanics · Sliding stiffness · Quartz minerals · Laboratory testing

## Introduction

The development of the discrete element method (DEM) by Cundall and Strack [1] in the 1970's and its improvements over the following decades [2–5] allowed the engineering mechanics research community to obtain insight into the behavior of particulate materials including granular soils. Complex mechanisms, such as, for example, creep phenomena in soils [6, 7], the interaction of particles and the effect of the frictional characteristics of soil grains [8] in their overall macro-scale response and energy dissipation mechanisms [9] have been better examined and understood.

In the simplest case, a linear elastic perfectly plastic law is implemented for particle contact response in sliding in DEM simulations. This simplified model is shown in Fig. 1 in which figure $K_T$ and $K_{T0}$ are the sliding (or tangential) stiffness and initial sliding stiffness, respectively, $F_T$ and $F_N$ are the sliding and normal forces at particle contacts, respectively. In the same figure, $\mu$ and $\mu_{st}$ denote the currently mobilized and static coefficients of inter-particle friction and $\delta$ is the sliding displacement. More complex models have been proposed over the past decades [10–12] for particle contact response in the sliding and normal directions (i.e., sliding force - displacement and normal force - displacement relationships). However, the available laboratory test data associated with experiments on real soil grains, which may be characterized by a sphere-sphere type of contact, which is non-conforming [13, 14] in nature, are limited. Previous research work and experimental effort has generally focused on the development of apparatuses investigating a flat block- flat block or a flat block-sphere type of contact [15, 16]. The importance of DEM analyses in soil mechanics research, the background associated with particle contact models and the necessity of laboratory research on real soil grain contacts have been thoroughly described in [17–19].

This brief note summarizes micromechanical experimental test results associated with the sliding stiffness at the contacts of real soil grains with a focus on the dependency of stiffness prior to the onset of a constant-state sliding on the normal force.

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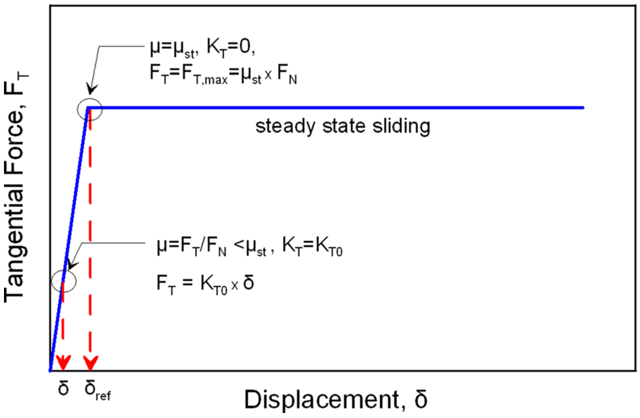


**Fig. 1** Linear elastic perfectly plastic force - displacement model frequently used as input in DEM for particle-contact response against sliding [18]

## Equipment, Materials and Methods

The custom-built micro-mechanical apparatus used in the experiments has been designed to test particles of regular convex and predominantly spherical shape, i.e., similar to a sphere with no re-entrant surfaces. Its development allows the investigation of frictional characteristics during sliding at the contacts of non-conforming surfaces, i.e., a sphere-sphere type of contact to be conducted, which is more applicable for soil mechanics purposes. Details of the apparatus have been thoroughly described by [17–19]. The mobilized friction angle and the sliding (or tangential) stiffness are examined at very small deformations, with shearing paths within a range of less than 1 μm to about 200–300 μm of sliding. This provides an insight into the force-displacement relationship at the contacts of soil grains prior to the onset of a steady-state sliding [18, 19].

One particle slides against the surface of a second-stationary particle through a horizontal system composed of a load cell, a linear micro-stepping motor and stiff mechanical parts. A desired vertical load at the contact of the particles is applied through a vertically positioned system. A scheme of the apparatus is given in Fig. 2. In this figure, a detail of two soil grains in contact of 2.2 mm in size is also shown; the picture was obtained from digital micro-cameras used in the experiments. It is noted that both fully saturated samples, immersed in water into a small cell and nominally dry particles tested at room temperature and humidity were included in the study.

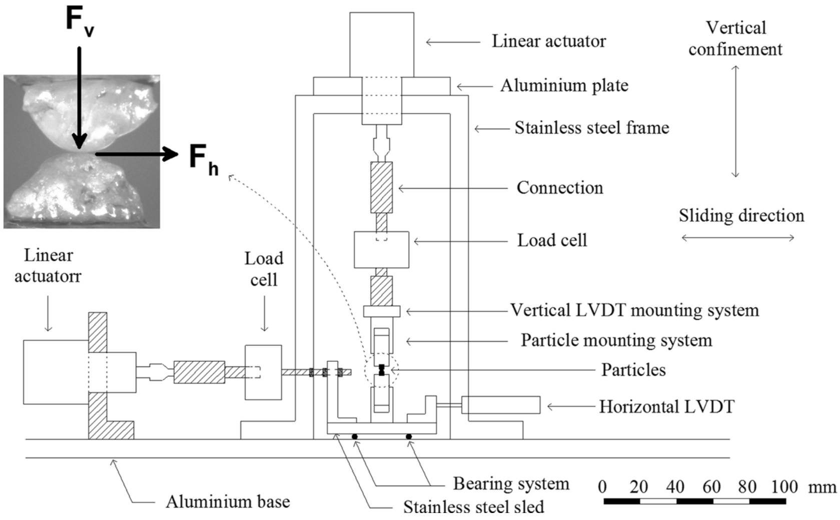
An attempt was made to examine the sliding stiffness due to repeating monotonic loading at the contacts of Leighton Buzzard sand (LBS) grains which are quartz particles of relatively spherical shape. The LBS particles were examined using an interferometry microscope and through measurements made on many particles it was found the average mean root square roughness ($S_q$) was equal to 0.38 μm with a standard deviation of ±0.19 μm [18]. It is noted that the use of relatively spherical particles in the experiments is necessary in order to resolve the horizontal and vertical forces and compute the normal and tangential to the sliding direction forces [17–19].

In order to minimize the electrical noise in the data and smooth the obtained force-displacement curves, a running average generally over about five to six data points was made. In addition, the sliding stiffness results presented in the study corresponded to tangent values calculated with a linear regression through a short section of the sliding load-deflection curve, typically using 10–30 data points in most experiments.

## Results and Discussion

Typical plots of sliding stiffness ($K_T$) against sliding displacement ($\delta$) for a pair of LBS particles and for a given normal

**Fig. 2** Scheme of micro-mechanical inter-particle loading apparatus and pair of quartz particles during an experiment (Note: Particles in contact have a diameter of 2.2 mm)



force ($F_N$=2 N) and repeated shearing tests following the same shearing path are given in Fig. 3. These results corresponded to quartz particles tested in a nominally dry state, i.e., the particles were cleaned, oven-dried and tested at room temperature and humidity. It is noted that the tests were carried out at low normal contact forces, in the range of 0.5 to 5 N. This is because previous research work has demonstrated through numerical simulations that this is the likely range of typical contact forces developed in sands at typical geotechnical engineering working loads [20, 21].

In this example the data showed a decrease of $K_T$ for the second shearing at very small displacements, in general less than 2 μm. $K_T$ was found equal to 0.36 N/microns for the first shearing and to about 0.20 N/microns for the second shearing. These values corresponded to the lowest displacement at which the sliding stiffness could be obtained, equal to about 0.5 μm. However, in previous experiments [18] the authors did not notice a systematic decrease of $K_T$ for repeating loading in all the particles tested.

In Fig. 3, as well as in previously published results by the authors [18, 19], it was observed that sliding stiffness degraded rapidly, with values almost equal to zero after a sliding displacement of about 3 to 5 μm. This is because of the high non-linearity in the force-displacement relationship at particle contacts. After a short shearing path, inter-particle sliding governs and shearing takes place at a constant force. For a sphere-flat block type of contact, Cole et al. [16] found in their monotonic sliding experiments on gneiss (grains from a metamorphic rock), values for the sliding stiffness from about 0.9 to about 1.4 N/micron, but these data corresponded to much higher normal forces in comparison to this study, with $F_N$ that ranged from about 7.5 to 11.5 N.

Repeating monotonic loading tests following the same shearing path at the contacts of a given pair of grains did not demonstrate a clear effect of the number of loading cycles on the mobilized inter-particle coefficient of friction (μ). It is noted that μ was derived from the ratio of tangential force over the normal force at any given displacement [19]. Typical results of sliding tests of a force-controlled type are shown in Fig. 4. These tests were performed at a rate of loading 5 and 10 N/h and at normal forces equal to 4 and 2 N respectively.

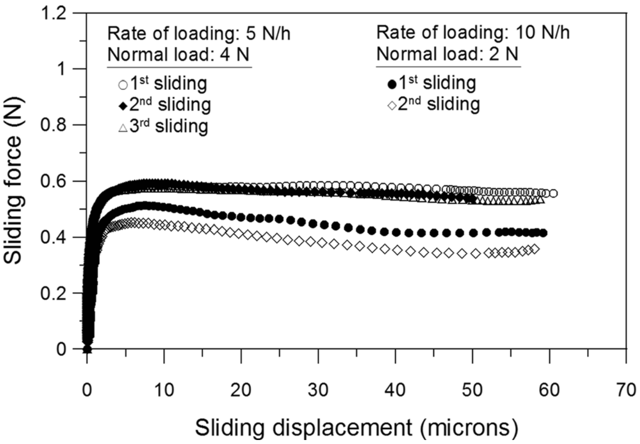


**Fig. 4** Typical plot of sliding force against sliding displacement at the contacts of LBS particles

From a set of 23 force-controlled experiments on different pairs of LBS particles of about 1 to 2 mm and 2 to 5 mm in size, and for rates of loading from 1.5 to 500 N/h, it was observed that there is a general trend of increasing values of sliding stiffness with an increase of $F_N$, but there was not observed any effect of the rate of loading on stiffness. The results of these tests are summarized in Fig. 5. It is observed that within the scatter of the data, there is almost a linear increase of stiffness with an increase of normal force, with a slope of 0.116 and a coefficient of correlation equal to 0.82. If we extended these data at larger $F_N$, for example 7 and 11 N, we would obtain values of $K_T$ equal to about 0.8 and 1.3 N/micron, which are reasonably close to the experimentally observed values by Cole et al. [16].

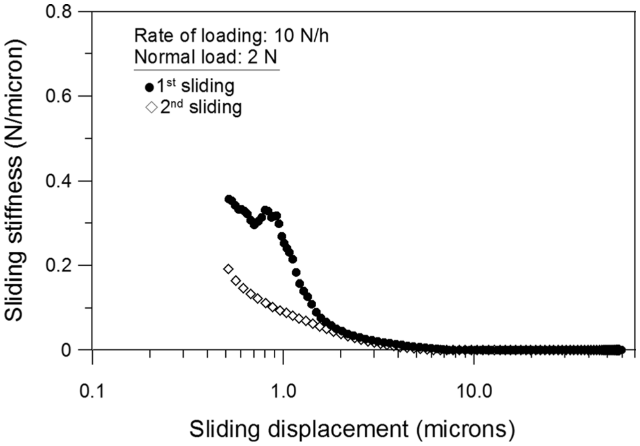


**Fig. 3** Typical plot of sliding stiffness at the contact of LBS particles at a normal load of 2 N and loading rate equal to 10 N/h

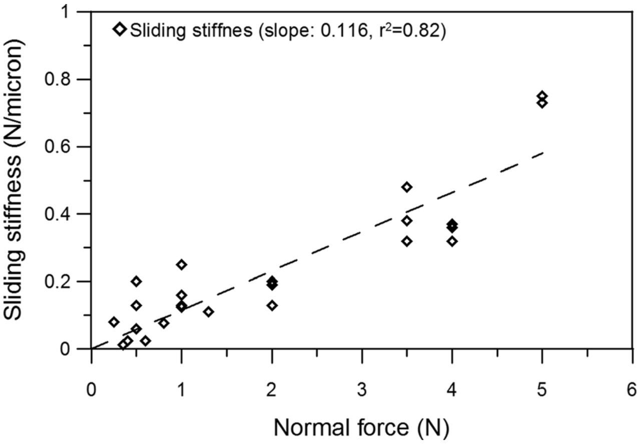


**Fig. 5** Sliding stiffness against normal force for LBS particles of 1.18–2.36 and 2.36–5.00 mm in diameter

## Conclusions

In this study, micro-mechanical inter-particle sliding tests were briefly reported with some trends with respect to the effect of repeating loading and the magnitude of the normal force ($F_N$) on the sliding stiffness at the contacts of quartz soil grains of relatively spherical shape. During the experiments, a steady-state sliding was observed at very small displacements, of the order of 2 to 5 μm. The magnitude of the normal force affected the sliding stiffness. It was observed, within the scatter of the data, that there was a linear increase of sliding stiffness with an increase of $F_N$. There was not observed a clear effect of repeating loading on the coefficient of friction or sliding stiffness, but experiments were limited to about 2 or 3 repeating shearing tests for a given pair of particles.

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