Effects of state of test sample, specimen geometry and sample preparation on dynamic properties of rubber–sand mixtures

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ABSTRACT: The study presents resonant column test results on sand mixed with granulated rubber. The experiments were carried out on samples within a range of rubber content from 0 to 15% by dry weight of sand–rubber mixtures, at variable isotropic effective stresses, p', state of test sample and sample preparation method. Both small-diameter specimens of 36 mm × 82 mm in size and large-diameter samples of about 70 mm × 140 mm in size were tested in torsional mode of vibration at very small shear strain amplitudes, in general less than 10^{-5}. Specimens were tested in a dry state, a moist state with a very small water content inclusion and a fully saturated state. The data indicated that for a given p', the increase of rubber content shifted the obtained resonant frequencies to lower values, the small-strain shear modulus, G_0, decreased, whereas small-strain material damping, D_{s0}, increased. The state of test sample, namely dry, moist or fully saturated, did not affect the obtained small-strain shear modulus values but did affect the obtained D_{s0}. G_0 could efficiently be described through typical formulas of G_0 against p' used for sands, by treating the volume of rubber particles as part of the total volume of voids into the void ratio function of the G_0–p' relationship. By preparing the samples at variable initial void ratios, comparisons could be obtained between samples of variable rubber contents and state of test sample but at a given resonant frequency. This is very important in capturing the behaviour of the mixtures in resonant column test which is a non-frequency-controlled method and thus the effect of rubber content and loading frequency could be de-coupled in the experiments.

KEYWORDS: Geosynthetics, Granulated rubber, Sand–rubber mixtures, Shear modulus, Damping ratio, Resonant column

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1. INTRODUCTION

Ground reinforcement by means of adding recycled rubber in soils has received great acceptance and applications the last two decades in geotechnical engineering projects. Laboratory research works have shown promising results for ground reinforcement–improvement by means of increased shear strength and overall satisfactory engineering performance. This is more pronounced in cases that the rubber content in the solid matrix is relatively low (Edil and Bosscher 1994; Masad *et al*. 1996; Tatlisoz *et al*. 1997; Zornberg *et al*. 2004). Edincliler *et al*. (2004) and Edincliler and Ayhan (2010) have proposed the use of tyre waste in ground reinforcement and improvement against static and dynamic loading in the form of fibre-shaped tyre buffing, whereas Fu *et al*. (2015) have shown that the granulated rubber inclusion in sand–rubber mixtures reduces sand grain breakage. The use of granulated rubber, pure or mixed with granular soils has been proposed for earthquake mitigation and improvement of soils and structures liquefaction resistance as well (Hazarika *et al*. 2008a, 2008b; Senetakis *et al*. 2009; Mavronicola *et al*. 2010; Pitilakis *et al*. 2010, 2011). Physical models, commonly shaking table experiments, and numerical studies have indicated promising results in providing the improved response of ground and structures against seismic wave propagation when recycled rubber aggregate was used (Hazarika *et al*. 2008a, 2008b; Uchimura *et al*. 2008).

The proper determination of the dynamic properties of geomaterials including soil–rubber mixtures, requires cyclic-dynamic element testing, for example, resonant column or cyclic triaxial tests. In particular, if one wishes to investigate the dynamic properties of geo-materials including both stiffness and attenuation, the resonant column may be considered as the standard and well-established experimental technique (ASTM D4015-92; Clayton, 2011). The accurate determination of small-strain stiffness (or alternatively elastic stiffness) and material damping is of high importance for the accurate modelling of the stress–strain curve and ground attenuation properties and thus in predicting the induced ground deformations due to wave propagation. This is because small-strain dynamic properties of soils are associated with the 'starting point' of the normalised stiffness degradation against shear strain curve (i.e. G/G_0 plotted against log γ) and the normalised damping ratio against shear strain curve (i.e. $D_s - D_{s0}$ plotted against log γ) (Jovicic and Coop 1997; Senetakis *et al.* 2012a, 2013). The small-strain dynamic properties and the normalised shear modulus and damping ratio curves of geomaterials are in turn important inputs in computer codes for seismic response analysis studies and dynamic design of foundations (Vucetic and Dobry 1991; Vucetic 1994; Darendeli and Stokoe 2001; Stokoe *et al.* 2004).

2. BACKGROUND ON DYNAMIC PROPERTIES OF SAND–RUBBER MIXTURES

Anastasiadis *et al.* (2012a) investigated the small-strain dynamic properties of dry sand–rubber and gravel–rubber mixtures examining the effects of the relative size of soil against rubber particles and the content of rubber aggregate in the mixtures in a resonant column apparatus. All experiments in that study were performed in samples of about 70 mm in diameter and 140 mm in length (denoted as large-diameter samples) and tested in the resonant column apparatus in a dry state. Anastasiadis *et al.* (2012b) presented experimental data derived from resonant column tests on samples of about 36 mm diameter and 82 mm length (denoted as small-diameter samples) of fully saturated sand–rubber mixtures and compared their results with dry large-diameter samples. In both studies (Anastasiadis *et al.* 2012a, 2012b), the samples, tested in a fully saturated or a dry state, were prepared using the dry compaction method in order to construct relatively dense specimens. The data of those two studies showed systematically a decrease of G_0 and increase in damping when the rubber content increased in the mixture. These observations were in agreement with the results by Feng and Sutter (2000) who examined the dynamic behaviour of sand–rubber mixtures in a resonant column apparatus. Previous research works by Kim and Santamarina (2008) and Pamukcu and Akbulut (2006) showed, for low rubber content, a slight increase of stiffness in the former study and a slight increase in both stiffness and damping in the latter study with an increase in rubber content. However, an opposite trend was observed for higher percentages of rubber, namely a decrease of stiffness and damping. The experiments by Kim and Santamarina (2008) were carried out in oedometer samples using the bender element method. It might be then some effect of the test method for the different trends observed for small rubber contents in the different studies, namely the research works by Anastasiadis *et al.* (2012a, 2012b) and Feng and Sutter (2000) in comparison with the research work by Kim and Santamarina (2008). This is because the resonant column method provides an average (or secant) stiffness of the granular assembly. In the bender element method the propagation of shear waves may be directed at preferable paths within the sample under consideration which might be more pronounced in binary mixtures with two different grain types of quite different elastic properties, such as quartz and rubber grains. Pamukcu and Akbulut (2006) tested their samples by adding a small amount of clay, while they compacted their specimens at a moisture content of about 6 to 7%. This might promote some more pronounced frequency effects to the laboratory test results because of the more pronounced viscous damping contribution in comparison, for example, with the dry samples tested by Anastasiadis *et al.* (2012a). A detailed review of the dynamic properties of sand–rubber mixtures observed in laboratory testing may be found in Anastasiadis *et al.* (2012a) or Senetakis *et al.* (2012a, 2012b). A review of the engineering properties of recycled rubber and rubber-reinforced soils including shear strength, dilation, compression and compaction properties and the classification of recycled rubber may be found in ASTM D6270-98, Edil (2004), Zornberg *et al.* (2004), Strenk *et al.* (2004), Humphrey (2008) or Edincliler and Ayhan (2010).

Past laboratory research studies examined the behaviour of composite soil–rubber mixtures with very limited information with respect to sample preparation effects (as for example in the study by Feng and Sutter (2000)). This means that most experimental research studies have adopted compaction methods (dry or moist) in order to construct relatively dense samples, which is more applicable for current practice for those materials. However, it is questionable whether, for example, for a given rubber content, isotropic effective stress, p', and state of test sample, a relatively dense and a relatively loose sand–rubber sample would exhibit similar small-strain damping ratio values. Because of the viscous nature of rubber grains, the inclusion of a given rubber content in a sand matrix may provide different quantitative results and interpretations, for example, due to specimen size effects (geometry effects) which in turn affect the range of frequencies obtained in a resonant column test. These frequency effects might be not that significant for the obtained shear modulus, but it is expected to affect material damping derivations to some extent. On the other hand, the resonant column method may provide an excellent indication for the average (secant) stiffness and attenuation properties of a granular assembly, for example a pure sand or a composite geomaterial such as sand–rubber mixtures. However, a comparison of resonant column experiments of sand–rubber mixtures for variable rubber contents at very similar resonant frequencies would provide some stronger indication for the actual effect of rubber inclusion in the solid matrix attenuation properties. This is because the inclusion of rubber grains affects the resonant frequencies obtained, along with some viscous damping component(s) added to the solid matrix of the mixtures.

The present study explored the small-strain dynamic properties of G_0 and D_{s0} of sand–rubber mixtures in a fixed-free resonant column apparatus. The database analysed was enhanced with additional test results previously published by the authors (Anastasiadis *et al.* 2012a, 2012b). The dynamic properties of the mixtures were investigated by examining the following effects: (*a*) granulated rubber inclusion and content; (*b*) isotropic effective stress, p'; (*c*) state of test sample, i.e. dry, slightly moist or fully saturated samples; (*d*) geometry of specimens by examining 'large-diameter samples' (70 mm diameter and 140 mm high) and 'small-diameter samples' (36 mm diameter and 82 mm high); (*e*) sample preparation method adopting the dry compaction and moist compaction methods for relatively dense samples and the hand-spooning method for relatively loose samples.

3. MATERIALS AND METHODS

3.1. Sand and rubber aggregate fractions

A medium-grained poor-graded sand of river origin was used as a physical portion of the mixtures composed of, dominantly, quartz particles. The sand has a mean grain size (d_{50}) of 0.56 mm and a coefficient of uniformity (C_u) of 2.76. Two fractions of a granulated rubber were used. For the large diameter samples of the study a coarse-grained poor-graded rubber with d_{50} = 2.80 mm and C_u = 2.29 was used and for the small-diameter samples a medium-grained poor-graded rubber was used with d_{50} = 0.40 mm and C_u = 2.65. Tests for the determination of the specific gravity of solids (ASTM D854-02) provided G_s values equal to 2.67 for the sand and 1.10 for both fractions of the granulated rubber (Anastasiadis *et al.* 2012a). The grading curves of the sand and the two granulated rubber fractions are shown in Figure 1. Optical microscope images of the sand and rubber grains are given in Figure 2 and an image of the coarse-grained rubber fraction is provided in Figure 3. The basic characteristics of the materials of the study including maximum grain size (D_{max}) are summarised in Table 1. It should be noted that for both small and large diameter samples, the maximum size of sand and rubber was kept within the typical limits, namely smaller than one-sixth of the sample diameter.

3.2. Sample preparation and experimental procedures

In total, 14 samples were prepared in the laboratory and tested in a resonant column of a fixed-free type (Drnevich 1967) in torsional mode of vibration in the range of very small deformations, in general less than 10^{-5}. Six of these samples have been presented by Anastasiadis *et al.* (2012a, 2012b). The samples of the study and some

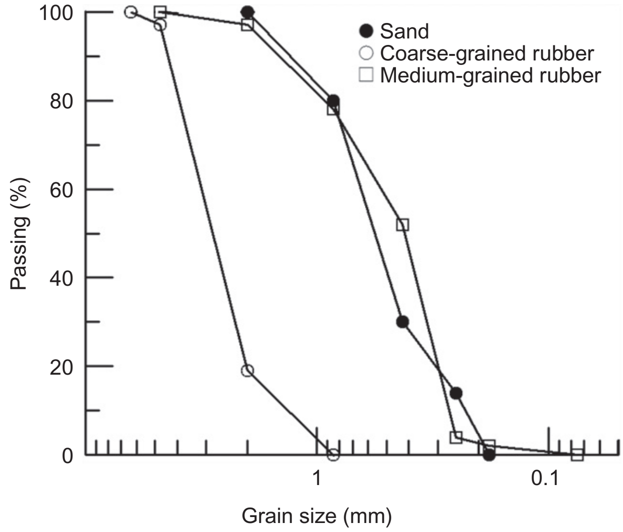

Figure 1. Grading curves of the materials of the study

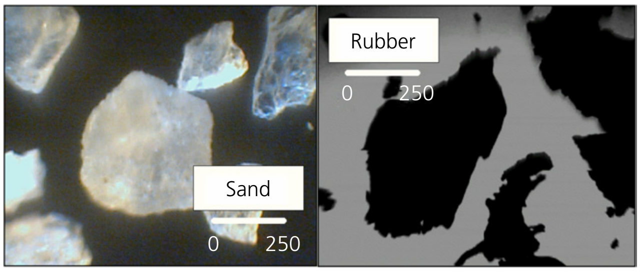

Figure 2. Optical microscope images of fine-grained portions (size 0.150–0.074 mm) of materials of study

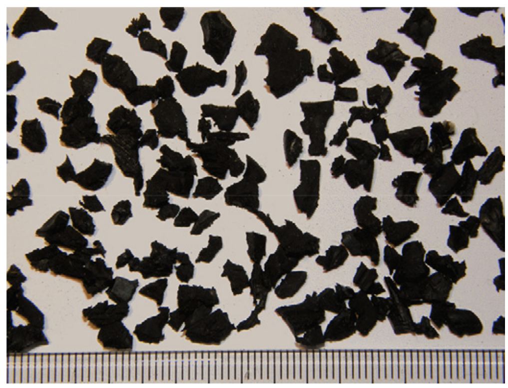

Figure 3. Image of coarse-grained rubber (note: scale in the image is expressed in mm)

basic characteristics, such as, for example, initial void ratio, state of test (i.e. dry, moist or fully saturated) and rubber content are summarised in Table 2. It is noted that the notation of the samples reflects the characteristics of the specimens and the state of the tests. In particular, the first term reflects the percentage of rubber, for example 'SRM15' and 'SRM00' corresponded to samples with 15 and 0% of rubber content by dry weight of mixture, respectively. The second term reflects the size of the specimen, namely '7' for a 7 cm diameter specimen and '3' for a 3.6 cm diameter specimen. The third term reflects the sample preparation method, using 'D' for dry and moist

Table 1. Materials of the study

Laboratory material	Medium-grained sand	Coarse-grained granulated rubber	Medium-grained granulated rubber
G_s	2.67	1.10	1.10
D_{50} (mm)	0.56	2.80	0.40
C_u	2.76	2.29	2.65
C_c	1.23	1.18	0.85
D_{max} (mm)	2.00	6.35	4.75
USCS	SP	–	–
ASTM	–	Granulated rubber	Granulated rubber
Origin	River sand	Recycled shredded tyres	Recycled shredded tyres

Table 2. Dynamic resonant column testing programme

No.	Specimen code	State	Specimen diameter (mm)	Rubber content (%)	e_o	W (%)	γ_{LA} (%)	A (MPa)	a
1	SRM05-7-L-D	Dry	70	5	0.801	-	6.0×10^{-4}–7.4×10^{-4}	4.46	0.51
2	SRM10-7-L-D	Dry	70	10	0.712	-	5.5×10^{-4}–7.6×10^{-4}	2.34	0.62
3	SRM15-7-L-D	Dry	70	15	0.729	-	7.9×10^{-4}–8.3×10^{-4}	3.00	0.56
4	SRM05-7-D-D[a,b]	Dry	70	5	0.494	-	4.1×10^{-4}–7.5×10^{-4}	5.66	0.47
5	SRM10-7-D-D[a,b]	Dry	70	10	0.511	-	4.3×10^{-4}–5.8×10^{-4}	5.95	0.47
6	SRM15-7-D-D[a,b]	Dry	70	15	0.471	-	3.2×10^{-4}–4.6×10^{-4}	5.31	0.48
7	SRM00-7-D-D[a,b]	Dry	70	0	0.588	-	4.8×10^{-4}–6.1×10^{-4}	4.86	0.51
8	SRM00-7-L-D	Dry	70	0	0.682	-	6.1×10^{-4}–8.8×10^{-4}	5.16	0.45
9	SRM15-7-D-S	Saturated	70	15	0.496	22.1	5.1×10^{-4}–6.6×10^{-4}	4.80	0.50
10	SRM00-7-D-S	Saturated	70	0	0.623	23.3	3.0×10^{-4}–4.4×10^{-4}	4.68	0.49
11	SRM15-7-D-M1	Moist	70	15	0.454	1.0	3.1×10^{-4}–9.4×10^{-4}	3.61	0.52
12	SRM15-7-D-M2	Moist	70	15	0.608	2.0	3.8×10^{-4}–1.1×10^{-3}	3.24	0.57
13	SRM15-3-D-S[b]	Saturated	36	15	0.562	23.1	5.4×10^{-3}–1.2×10^{-2}	1.17	0.71
14	SRM00-3-D-S[b]	Saturated	36	0	0.623	23.3	2.8×10^{-3}–4.0×10^{-3}	3.16	0.54

[a]Anastasiadis *et al.* (2012a). [b]Anastasiadis *et al.* (2012b).

compacted dense specimens and 'L' for loosely constructed specimens adopting the hand-spooning method. Finally, the fourth term reflects the state of the test, by using 'D' for samples tested in a dry state, 'S' for samples tested in a fully saturated state and 'M1' or 'M2' correspond to samples prepared with moist compaction at a water content of 1 or 2%, respectively.

Samples were tested at different isotropic effective stresses, p', in general within a range of 25 to 400 kPa with a sequence of three to five increments of p', in general equal to 25, 50, 100, 200 and 400 kPa. Details of the preparation procedure for dense samples (i.e. dry compaction, experiments in dry or fully saturated state), the equipment used in the experiments and the analysis-interpretation of the resonant column test results have been presented thoroughly by Anastasiadis *et al.* (2012a, 2012b) and Senetakis *et al.* (2012a, 2012b, 2012c). For the moist-compacted samples the preparation procedure was identical to the procedure used for dry-dense samples with the addition of a target moisture content prior to the construction of the specimens in the resonant column apparatus. For the loose samples prepared with the hand-spooning method, similar procedure as the one presented by Senetakis *et al.* (2012c) for dry and loose sands was adopted.

3.3. Resonant column test data analysis

For the analysis of the torsional resonant column test results the ASTM D4015-92 specification was adopted. Based on the formulas from the theory of elasticity (Richart *et al.* 1970) and the correlation between shear wave velocity, V_s, and shear modulus, G, (Equation 1), small-strain stiffness was computed through Equation 2 as a function of the resonant frequency, f_T, the length, L, and the mass density, ρ, of the specimen (ASTM D4015-92).

$$G = \rho \times V_s^2 \tag{1}$$

$$G = \rho \times (2 \times \pi \times L)^2 \times \left(\frac{f_T}{F_T}\right)^2 \tag{2}$$

In Equation 2, F_T is a non-dimensional frequency factor (ASTM D4015-92) which is calculated as a function of the polar mass moment of inertia of the attached mass on top of the sample, denoted as J_A, and the polar mass moment of inertia of the specimen (denoted as J), which is computed from the geometry of the sample. Variable J_A is derived from the calibration test (ASTM D4015-92) and J is equal to $M \times d^2/8$, where M and d are the weight and diameter of the sample, respectively.

The small-strain damping ratio was estimated using the steady-state method (ASTM D4015-92). The shear strain amplitude was estimated as an effective shear strain which was considered equal to 0.8 times the maximum shear strain at the perimeter of the specimen (Chen and Stokoe

1979; ASTM D4015-92). The maximum shear strain was estimated from the recorded displacement of the specimen on its top from an accelerometer attached to the drive mechanism.

Records of sample length changes during isotropic compression or during resonant column tests were implemented through a vertically positioned linearly variable differential transformer (LVDT) attached on top of the sample. For the saturated specimens, volume changes were recorded from a volume change/back pressure controller of GDS type. For dry or moist samples, the volumetric strain (ε_v) was estimated from the LVDT records assuming isotropic compression and thus Equation 3 was used, in which ε_a is the axial strain.

$$\varepsilon_v = 3 \times \varepsilon_a \quad (3)$$

4. RESULTS AND DISCUSSION

4.1. Axial deformations and void ratio changes of samples during isotropic compression

As a general trend observed in the study, the addition of rubber in the mixtures increased the axial strain (ε_a) recorded by the vertically positioned LVDT during the increase of the isotropic effective stress, p'. Typical plots of axial strain against p' are depicted in Figure 4. For example, at $p' = 100$ kPa, the pure sands (dense and loose samples) had ε_a in general less than 0.15% in magnitude, whereas a dense sample with 15% of rubber by mixture weight had almost twice the axial strain at that level of p'. It is noted that while dense and loose samples of the pure sand had quite similar ε_a values at a given p', the preparation method affected the observed axial strains of the mixtures during the increase of the isotropic stress markedly. For example, as shown in Figure 4, the loose sample with 15% rubber content (SRM15-7-L-D) had almost five times larger axial strain at $p' = 100$ kPa in comparison with the dense sample with the same content of rubber (SRM15-7-D-D). It should be noted that the axial strains in Figure 4 corresponded to cumulative strains during the increase of p' expressed in percentile scale. However, the overall effect of p' in the changes of void ratio were relatively small. Typical variations of void ratio (e) against p' are given in Figure 5. For the range of isotropic stresses of the study, void ratio changes were in general less than 5% for an increase of p' from 25 to 400 kPa and for most samples less than 2%.

4.2. Typical range of resonant frequencies

A typical range of resonant frequencies (f_T) obtained during the tests against p' for representative samples is depicted in Figure 6. For a given sample size and preparation method, for example dry-compaction at a given energy, the addition of rubber decreased the resonant frequency obtained in the experiments, as for example comparing specimens SRM15-7-D-D (15% rubber, dense-dry sample) and SRM00-D-D (pure sand, dense-dry sample). For a given rubber content, sample size and preparation method, samples tested in a dry or fully saturated state had very similar f_T values, as for example comparing specimens SRM15-7-D-D (15%, dense-dry sample) and SRM15-7-D-S (15% rubber, dense-fully saturated sample). The samples prepared with moist-compaction had very similar resonant frequencies with samples tested in a dry or fully saturated state, but it should be considered that suction effects might be not that significant since the addition of moisture was only 1 to 2%. The effect of specimen geometry was significant in the obtained resonant

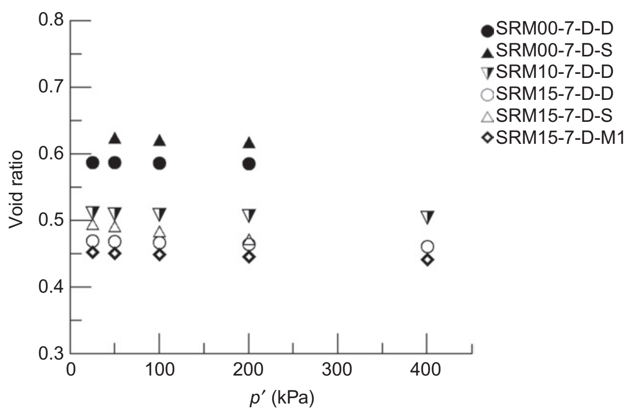

Figure 5. Void ratio against p' for typical samples

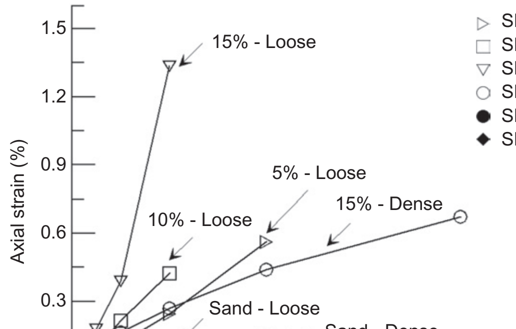

Figure 4. Axial strain against p' for typical samples

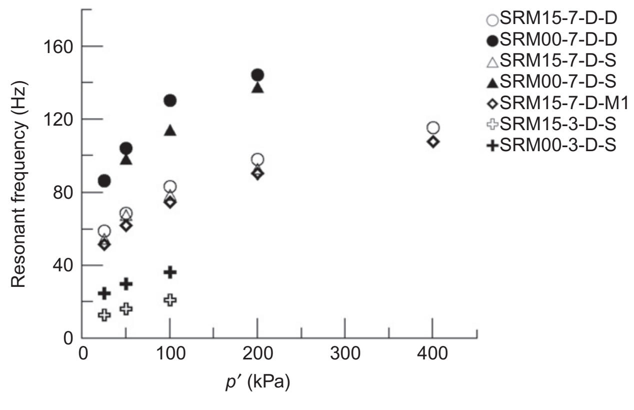

Figure 6. Typical resonant frequencies measured for sand and sand–rubber samples against p'

frequencies, with much lower f_T values for the small-diameter samples in comparison to the large-diameter specimens. Small-diameter samples were tested in a range of f_T from about 10 to 35 Hz, whereas large-diameter samples were tested in a range of f_T from about 45 to 150 Hz. For a given specimen, the obtained resonant frequencies increased with an increase in p' which reflects the stiffer solid skeleton at higher confining pressures.

4.3. Small-strain shear modulus

For a given sample size, the observed trends for the resonant frequencies as demonstrated in Figure 6 were qualitatively identical and for the corresponding small-strain shear modulus trends as shown in Figure 7 which presents plots by means of G_0 against p', where G_0 is the small-strain shear modulus. A comparison between samples SRM15-7-D-D and SRM00-D-D or samples SRM15-7-D-D and SRM15-7-D-S indicated that the pure sand had much higher G_0 values than the mixture with 15% rubber. Dense and saturated samples for a given rubber content, preparation method and p' had almost identical G_0. On the other hand, small-diameter samples had much lower values of resonant frequencies than large-diameter samples, but G_0 was not affected by specimen geometry. This is because the effect of specimen geometry was eliminated in the wave propagation equation (Equation 2) as both the obtained resonant frequency and the polar mass moment of inertia were affected by sample geometry.

In Figure 8, G_0 of variable samples have been plotted normalised with respect to a void ratio function, $F(e)$, against p'. The $F(e)$ function used in the study has been proposed by Jamiolkowski *et al.* (1991) for sands and the void ratio (e) or, more precisely, the sand–rubber solid matrix void ratio was used in this case, in which the volume of solids included both sand and rubber particles defined by Equation 4. The void ratio function is expressed through Equation 5.

$$e = \frac{V_{\text{voids}}}{V_{\text{sand}} + V_{\text{rubber}}} \tag{4}$$

$$F(e) = \frac{1}{e^{1.3}} \tag{5}$$

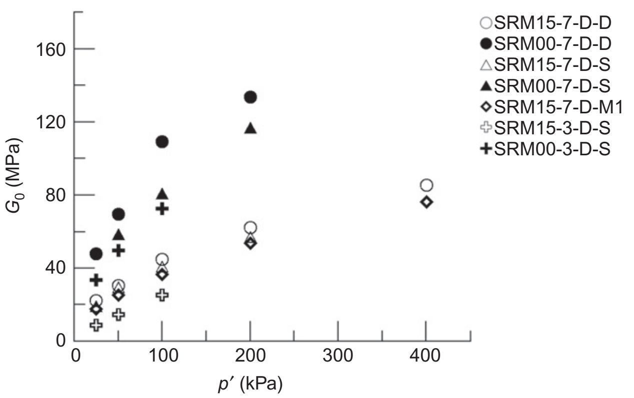

Figure 7. Typical plots of small-strain shear modulus for sand and sand–rubber samples against p'

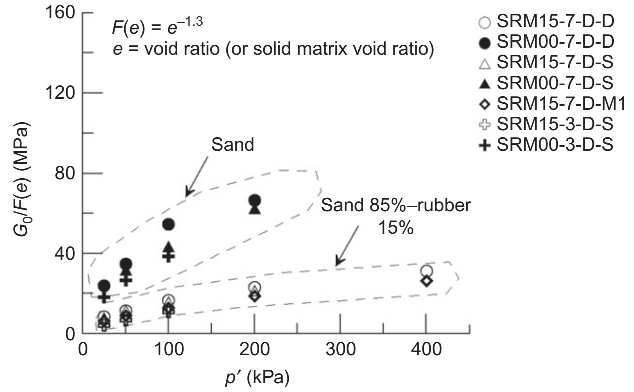

Figure 8. Typical plots of normalised small-strain shear modulus for sand and sand–rubber samples against p' (note: the sand–rubber matrix void ratio is used into the void ratio function)

In Equation 4 V_{voids} is the volume of voids within the sample, V_{sand} is the volume of sand particles and V_{rubber} is the volume of rubber particles.

For a given rubber content, the void ratio function works efficiently by means of eliminating effects such as preparation method and initial void ratio, but for a given p' the normalised $G_0/F(e)$ values were found to be much higher for the sand than the sand–rubber mixtures. This is because the contribution of rubber grains, even though it is significant in the formulation of the solid matrix, for example by reducing the sand–rubber matrix void ratio (Anastasiadis *et al.*, 2012a), is considered very small to negligible for the solid matrix stiffness. This is associated with the high deformability of rubber particles. The coupled effect of rubber inclusion and content results in (*a*) reduction of sand–rubber solid matrix void ratio and (*b*) negligible contribution on sand–rubber matrix stiffness. This coupled effect has been analytically expressed in the past through an equivalent void ratio, e_{eq} (Feng and Sutter 2000; Anastasiadis *et al.* 2012a, 2012b), which was very similar to the granular void ratio that has been used, for example, for silt–sand mixtures. In this case, the volume of rubber solids was considered as part of the total volume of voids because their contribution to the overall sand–rubber solid matrix stiffness is relatively negligible.

The results of the normalised small-strain stiffness adopting the equivalent void ratio into the same void ratio function used for the results of Figure 8, are presented in Figure 9. The equivalent void ratio in this case is expressed through Equation 6 and thus the void ratio function $F(e)$ of Equation 5, corresponds to an equivalent void ratio function in this case, denoted as $F(e_{\text{eq}})$ expressed through Equation 7.

$$e_{\text{eq}} = \frac{V_{\text{voids}} + V_{\text{rubber}}}{V_{\text{sand}}} \tag{6}$$

$$F(e_{\text{eq}}) = \frac{1}{e_{\text{eq}}^{1.3}} \tag{7}$$

In this case, the results are much less scattered (comparison between Figures 8 and 9), and it may be considered

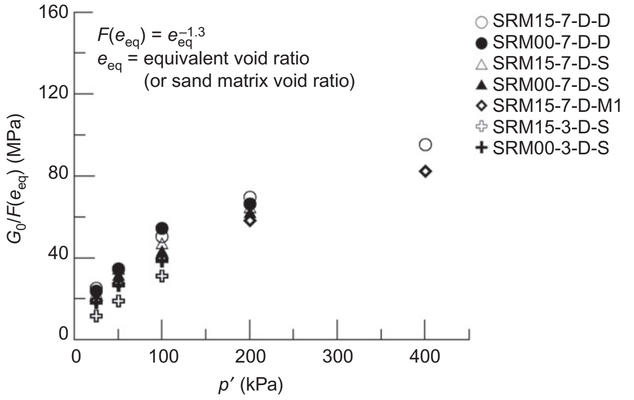

Figure 9. Typical plots of normalised small-strain shear modulus for sand and sand–rubber samples against p' (note: the equivalent or sand-matrix void ratio is used into the void ratio function)

that the void ratio function proposed by Jamiolkowski *et al.* (1991) for sand works efficiently for sand–rubber mixtures as well when an equivalent void ratio that corresponds to the sand-matrix void ratio is used.

The small-strain shear modulus of granular assemblies has been expressed analytically as a function of isotropic effective stress, p', and void ratio through Equation 8. In this equation, A is a material constant and the exponent (α) expresses the effect of p' on G_0 with typical values about 0.5 for most sands. This exponent reflects the effect of p' in fabric changes within the granular assembly and it is associated as well with the micro-mechanisms that take place at particle-to-particle contacts, which in turn affect constant-state properties such as small-strain shear modulus (Cascante and Santamarina 1996).

$$G_0 = A \times F(e) \times \left(\frac{p'}{1}\right)^{\alpha} \quad (8)$$

In this study, adopting the equivalent void ratio defined in Equations 6 and 7, Equation 8 for sand–rubber mixtures may be re-written as

$$G_0 = A \times F(e_{eq}) \times \left(\frac{p'}{1}\right)^{\alpha} \quad (9)$$

Through a best-fit of the experimentally derived small-strain moduli by plotting $G_0/F(e_{eq})$ against p' and by adopting a power-law fitting curve for each sample (Santamarina and Cascante 1998), the constant A and the exponent (α) were determined. These values are summarised in Table 2. With the exception of sample SRM15-3-D-S, which had a much lower A value and much higher (α) value in comparison to the remaining thirteen specimens of the study, the small-strain shear modulus constants ranged between 2.34 and 5.95 (MPa) for the constant A and between 0.47 and 0.62 for the exponent (α). The obtained coefficients of correlation (r^2) of the power-law fitting curves were found in a range of 0.99 and 1.00. It is noted that the constant A of Equation 8 is expressed in MPa (i.e. same units as G_0). It is also noted that within the range of rubber content used in the study, namely between 0 and 15%, no significant effect of rubber content on the magnitude of the exponent (α) was observed.

4.4. Small-strain material damping

Typical plots of small-strain material damping (D_{s0}) against p' are depicted in Figure 10. There was observed a general trend of decreasing D_{s0} values with an increase of p' no matter the content of rubber in the mixtures. A comparison between samples SRM00-7-D-D (dry-dense pure sand) and SRM00-7-L-D (dry-loose pure sand) implied that for a given level of p', D_{s0} values of the pure sand were almost independent on void ratio. Previous studies on granular soils have shown very similar qualitative trends, for example the studies by Senetakis *et al.* (2012c) and Senetakis *et al.* (2013). For the pure sandy samples tested in a dry state, D_{s0} values were found to be less than unity expressed in percentile scale. The results of Figure 10 indicated an increase of material damping with an increase in rubber content.

In Figure 11, the results of D_{s0}–p' for five samples of sand–rubber mixtures with 15% of rubber content are summarised. These samples were tested at variable states,

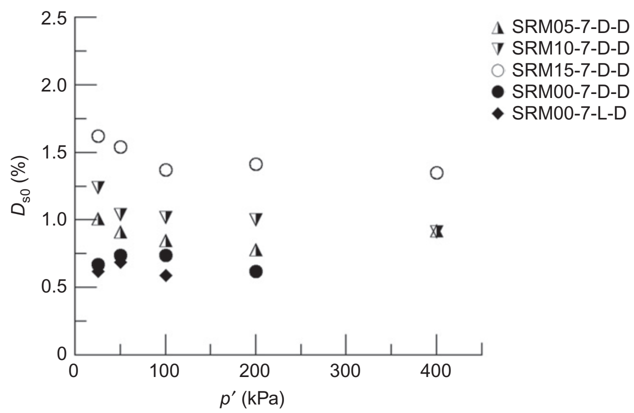

Figure 10. Typical plots of small-strain material damping for sand and sand–rubber samples against p'

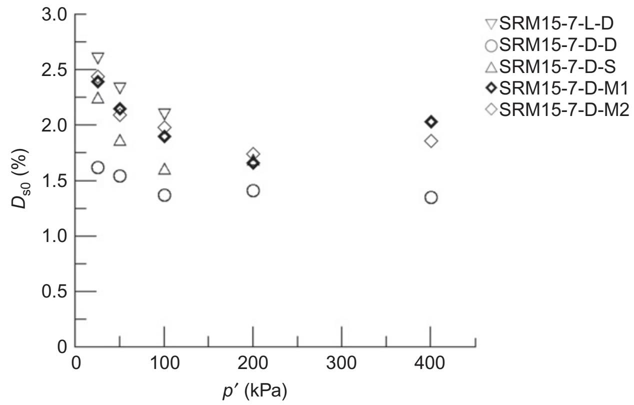

Figure 11. Small-strain material damping of mixtures with 15% rubber prepared with different preparation methods and tested at variable states

namely dry, moist at water content of 1 and 2% and fully saturated. The dense sample tested in a fully saturated state (SRM15-7-D-S) had slightly higher values of D_{s0} in comparison with the dry-dense sample (SRM15-7-D-D), whereas the two samples tested at a moist state (SRM15-7-DM1 and SRM15-7-D-M2) had slightly higher D_{s0} values than the fully saturated sample. These observations were attributed primarily to the additional viscous damping component in moist and saturated samples which increased material damping in comparison with the dry specimens. These results are in qualitative agreement with the observed response of pure sands, for example the research work by Madhusudhan and Kumar (2013). In their study, the effect of the degree of saturation on material damping of a quartz sand was investigated. Interestingly, the dry sample tested at a relatively loose state had much higher values of damping than the dry sample tested in a dense state (Figure 11).

The increase of rubber content in the mixtures shifted the obtained resonant frequencies to lower values (as shown for example in Figure 6). Thus, one would argue that the effect of rubber content in the observed trends associated with small-strain material damping may be attributed, primarily, to frequency effects because the rubber particles were viscous in nature. An advantage of testing the samples at variable initial void ratios through different preparation methods was that comparisons could be obtained between samples of different rubber contents but at relatively similar resonant frequencies. This overcame the shortcoming of the resonant column method in that the frequency at which a sample was tested and the dynamic properties were obtained could not be controlled, such as, for example, in cyclic triaxial or torsional shear tests. An example is given in Figure 12, in which the material damping values of a dense-dry sample with 15% rubber by mixture weight are compared with corresponding D_{s0} values derived from a loose-dry sample with 5% rubber by mixture weight. In this figure, D_{s0} values are plotted against the resonant frequency at variable levels of p', and the p' values are also depicted in the figure. The plots of these two samples provide comparisons at a given level of p' and resonant frequency (f_T), as material damping values for those specimens were obtained at about the same resonant frequency and thus frequency effects were eliminated. It is clear in Figure 12 that material damping was strongly affected by rubber content with higher D_{s0} values for the sample with higher rubber content.

The D_{s0} values of three samples with 15% rubber but variable states of test sample, namely dry, moist and fully saturated, are plotted against f_T in Figure 13. In this figure, the values of p' at which material damping was obtained are also depicted. It can be observed that for given p', the resonant frequencies of the three samples were almost the same, and thus the effect of the degree of saturation and state of test on material damping did not include significant frequency effects in the obtained D_{s0} values. The results in this figure clearly demonstrated that the higher values of material damping for fully saturated and moist samples in comparison with dry samples were exclusively due to the additional component of viscous damping.

5. CONCLUDING REMARKS

The torsional resonant column tests performed in this research work on sand–rubber mixtures enhanced with previous published data by the authors led to following basic conclusions.

- The increase of rubber content in the sand–rubber mixtures decreased the small-strain shear modulus, G_0, and increased the small-strain material damping, D_{s0}.
- The obtained resonant frequencies during dynamic loading were shifted to lower values with an increase of rubber content. However, a comparison for example, of D_{s0} values of samples of variable rubber contents but tested at almost the same resonant frequency, indicated that the qualitative resonant column observations associated with the effect of rubber content on material

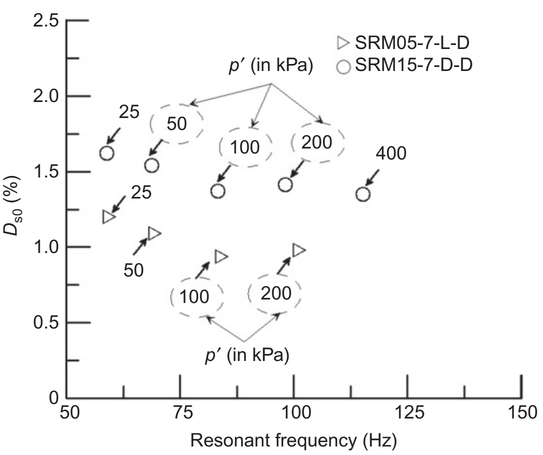

Figure 12. Comparison between small-strain material damping of mixtures with 5 and 15% rubber at about the same p' and resonant frequencies

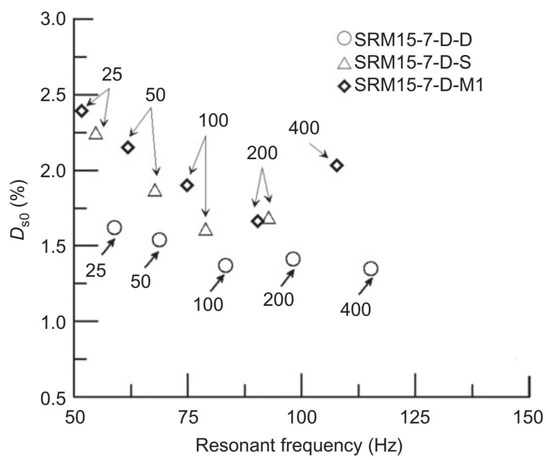

Figure 13. Comparison between small-strain material damping of samples with 15% rubber tested at variable states (numbers in the figure denote p' expressed in kPa)

damping are rigid and independent of possible viscous damping-frequency effects. The inclusion of samples of variable rubber content tested at variable initial densities overcame the disadvantage of the resonant column method, in that it is not a frequency-controlled experimental technique. Thus, in the study, comparisons could be made between samples of variable rubber contents but tested at almost the same resonant frequency.

- The effect of the state of the test sample, namely dry, fully saturated or moist and for a given content of rubber affected material damping but had an almost negligible effect in the obtained small-strain shear moduli. The additional viscous component of damping in fully saturated samples increased D_{s0} values in comparison with the obtained values derived from dry samples with the additional effect of the degree of saturation and these observations were in agreement with previous studies on pure sands.
- Specimen size did not affect the obtained small-strain shear modulus even though it did affect the obtained resonant frequencies.
- Typical G_0–p' relationships proposed in the literature for sands and for the range of rubber contents of the study can efficiently be used to describe the G_0–p' relationship of sand–rubber mixtures by treating the volume of rubber particles as part of the total volume of voids. In this case, an equivalent void ratio function was used.

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NOTATION

Basic SI units are given in parentheses.

A	small-strain shear modulus constant (Pa)
C_c	coefficient of curvature (dimensionless)
C_u	coefficient of uniformity (dimensionless)
d	specimen diameter (m)
d_{50}	mean grain size (m)
d_{max}	maximum grain size (m)
D_s	damping ratio (dimensionless)
D_{s0}	small-strain damping ratio (dimensionless)
e	void ratio (dimensionless)
e_o	initial void ratio (dimensionless)
e_{eq}	equivalent void ratio (dimensionless)
f_T	resonant frequency (Hz)
F_T	non-dimensional frequency factor (dimensionless)
$F(e)$	void ratio function (dimensionless)
$F(e_{eq})$	equivalent void ratio function (dimensionless)
G	shear modulus (Pa)
G_0	small-strain shear modulus (Pa)
G_s	specific gravity of solids (dimensionless)
J	polar mass moment of inertia of specimen (kg × m^2)
J_A	polar mass moment of inertia of attached mass on top of specimen (kg × m^2)
L	length of specimen (m)
M	mass of specimen (kg)
p'	isotropic effective stress (Pa)
V_{voids}	volume of voids (dimensionless)
V_{rubber}	volume of rubber particles (dimensionless)
V_{sand}	volume of sand particles (dimensionless)
V_s	shear wave velocity (m/s)
w	water content (dimensionless)
α	small-strain shear modulus constant (dimensionless)
γ	shear strain amplitude (dimensionless)
γ_{LA}	small-strain shear strain obtained in the experiments (dimensionless)
ε_a	axial strain (dimensionless)
ε_v	volumetric strain (dimensionless)
ρ	mass density of specimen (kg/m^3)

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