Geotechnical Testing Journal, Vol. 35, No. 6, 2012
Available online at www.astm.org
doi:10.1520/GTJ20120073

Kostas Senetakis,[1] *Anastasios Anastasiadis,*[2] *and Kyriazis Pitilakis*[2]

The Small-Strain Shear Modulus and Damping Ratio of Quartz and Volcanic Sands

REFERENCE: Senetakis, Kostas, Anastasiadis, Anastasios, and Pitilakis, Kyriazis, "The Small-Strain Shear Modulus and Damping Ratio of Quartz and Volcanic Sands," *Geotechnical Testing Journal*, Vol. 35, No. 6, 2012, pp. 1–17, doi:10.1520/GTJ20120073. ISSN 0149-6115.

ABSTRACT: The dynamic properties of soils in the region of very small strains are essential for any seismic design. This paper aims to investigate the dynamic small-strain shear modulus (G_O) and damping ratio (D_O) of reconstituted dry sands of variable mineralogy, shape, and grain-size distribution. In particular, the low-amplitude torsional resonant column test results of 31 specimens are synthesized, 19 specimens of natural and quarry sands predominately composed of quartz particles, and 12 specimens of volcanic sands composed of rhyolitic glassy rock of porous particles. It is concluded that the volcanic sands exhibit significantly lower G_O values and slightly lower D_O values in comparison to the quartz ones whilst the response of the quartz sands is significantly affected by the shape of the particles. The differences in the observed responses between quartz and volcanic sands are partially attributed to the variability in particles density, morphology, and mineralogy, as well as the higher void ratio and the lower dry density that the volcanic sands exhibit in comparison to the quartz ones. Overall, the effects of the mean effective confining pressure ($\sigma_m{}'$), the void ratio (e), and the grain-size distribution on the dynamic response of the volcanic soils follow a similar trend as in the quartz sands. Using the general form of available relationships presented in the literature, and after modifying the "constant" parameters, appropriate equations, stemming from the low-amplitude resonant column data test, are proposed that may be used for the estimation of the small-strain shear modulus and damping ratio separately for natural quartz sands, quarry quartz sands, and volcanic granular soils.

KEYWORDS: resonant column testing, shear modulus, damping ratio, quartz sands, volcanic soils

Introduction

Accurate predictions of soil deformations and earthquake-induced loads on structures and geo-structures require the knowledge or the realistic estimation of the dynamic properties of soils. In current geotechnical engineering practice, the dynamic properties of soils are often expressed in terms of the shear modulus, G, and the damping ratio in shear, D. The shear modulus is correlated to the shear wave velocity, V_s, and the bulk density, ρ, through Eq 1 and expresses the stiffness, whereas the damping ratio expresses the absorbing capacity of a soil subjected to cyclic loading (Richart et al. 1970; Hardin and Drnevich 1972a, 1972b; Gazetas 1991; Ishihara 1996).

$$G = \rho \times V_s^2 \tag{1}$$

In the region of small strains, or more precisely "very small strains" following Atkinson and Sallfors (1991) or Clayton (2011), on the order of $10^{-4}\%$ to $5 \times 10^{-3}\%$ depending on the type of the soil and the confining pressure level, it may be assumed that soils exhibit approximately linear-elastic behavior. In this region, the shear modulus is practically constant and corresponds to its maximum value, often expressed as small-strain or initial shear modulus, G_O, whereas the damping ratio corresponds to its minimum value, often expressed as small-strain or initial damping ratio, D_O. As the shearing strain amplitude exceeds a certain value, namely the elastic or linear threshold strain, γ_t^e or γ_t^1 (Vucetic 1994; Ishihara 1996), soils exhibit non-linear behavior; in this region, the shear modulus decreases and the damping ratio increases with increasing shearing strain. On a micro-scale point of view, beyond the elastic threshold strain, slippage at particle contacts occurs, the internal fabric of soils changes and thus, the value of γ_t^e strongly depends on the magnitude of the normal forces developed at the particle contacts, the inter-particle coefficient of friction and the stiffness of the particles (Santamarina et al. 2001).

The small-strain shear modulus, G_O, of granular soils is strongly affected by the mean effective confining pressure, σ_m', and the void ratio, e. G_O increases with increasing σ_m' primarily because of the development of normal forces of higher magnitude at particle contacts (Santamarina et al. 2001), and secondarily because of the denser packing of the solid matrix, whereas G_O decreases with increasing void ratio. Except for the mean effective confining pressure and the void ratio, past and recent studies have underlined the important effect of the grain size distribution, often expressed in terms of the mean grain size, D_{50} and the coefficient of uniformity, C_u (e.g., Iwasaki and Tatsuoka 1977; Ishihara 1996; Menq 2003; Menq and Stokoe 2003; Lontou and Nikolopoulou 2004; Hardin and Kalinski 2005; Wichtmann and Triantafyllidis 2009; Wichtmann et al. 2011) as well as the effect of particle shape, often expressed in terms of the mean roundness, R_m (e.g., Hardin and Richart 1963; Edil and Luh 1978; Cho et al. 2006) on the small-strain shear modulus, G_O, of granular soils. The effects of the morphology and mineralogy of the particles are

Manuscript received October 10, 2011; accepted for publication July 21, 2012; published online September 2012.
[1]Dept. of Civil and Architectural Engineering, City Univ. of Hong Kong, Hong Kong, China (Corresponding author), e-mail: ksenetak@cityu.edu.hk
[2]Dept. of Civil Engineering, Aristotle Univ. of Thessaloniki, Thessaloniki, Greece.

also important on the cyclic response of granular soils because these factors are strongly correlated to particles deformation and inter-particle contact stiffness (Clayton 2011), but currently most studies have been focused on the dynamic response of soils predominately composed of quartz particles.

With reference to sandy soils of mean grain size less than or equal to 2.00 mm, approximately, it is concluded that G_O increases with increasing D_{50}, decreasing C_u and decreasing R_m. On a micro-scale point of view and based on the available numerical results by Radjai et al. (1998), Wichtmann and Triantafyllidis (2009) correlated the aforementioned trend of increasing G_O with decreasing C_u with the development of strong and weak particle contacts on soils of high C_u, whereas in uniform soils only strong contacts are developed and thus the solid matrix is stiffer in the latter case. However, the overall effect of C_u on the small-strain shear modulus is fairly complex because of the additional effect of C_u on particles packing, which is often expressed in terms of the void ratio (Senetakis 2011); at a given compaction energy of an assembly of coarse particles, the increase of C_u leads to the decrease of the void ratio (Youd 1973; Santamarina et al. 2001). In addition, the increase of G_O with decreasing R_m is true for mean effective confining pressures above 20 to 25 kPa as will be discussed later in this paper.

For practical-design purposes, many researchers have correlated the small-strain shear modulus with the mean effective confining pressure and the void ratio through the general form of Eq 2. Hardin and Richart (1963), Shibata and Soelarno (1975), Iwasaki et al. (1978), Kokusho (1980), Yu and Richart (1984), Kanataki et al. (1994), Zhou and Chen (2005), and others proposed relationships of the general form of Eq 2, in which $n_P = 1$. Hardin (1978), Chung et al. (1984), Saxena and Reddy (1989), Jamiolkowski et al. (1991), and Wichtmann and Triantafyllidis (2009) proposed relationships of the general form of Eq 2, in which $n_P = n_G$ and, finally, Menq (2003) proposed a relationship of the general form of Eq 2 in which $n_P = 1$ and the mean effective confining pressure is expressed as σ'_m/P_a, where P_a is the atmospheric pressure.

$$G_O = A_G \times F(e) \times P_a^{1-n_P} \times \left(\sigma'_m\right)^{n_G} \qquad (2)$$

In Eq 2, A_G is a "constant" parameter that depends on soil type, n_G is an exponent that expresses the slope of the diagram G_O-σ'_m, and $F(e)$ is the void ratio function. Most researchers have proposed a value of the exponent n_G equal to 0.5. However, in recent studies, it was concluded that the aforementioned exponent is strongly affected by the coefficient of uniformity, C_u (e.g., Menq 2003; Wichtmann and Triantafyllidis 2009) and the mean roundness, R_m (e.g., Cho et al. 2006); n_G increases with increasing C_u and decreasing R_m.

Regarding the small-strain damping ratio (D_O) of granular soils, it is concluded that D_O slightly decreases with increasing σ'_m, whereas the effect of void ratio is not that important as in G_O (e.g., Laird 1994; Menq 2003; Zambelli et al. 2006; Senetakis 2011). Because of the difficulties relative to the accurate measurements in the laboratory of the damping ratio at very small strain levels where the background noise affects the experimental results (Menq 2003), the available empirical relationships for the estimation of D_O of granular soils are extremely limited and somehow the overall uncertainties on estimating the damping ratio of granular materials in the laboratory are more pronounced in comparison to the estimation of the shear modulus (Zambelli et al. 2006). In general, the small-strain damping ratio of dry clean sands and considering confining pressures above 20 kPa, is less than unity (e.g., Menq 2003; Senetakis 2011), whereas at high confining pressures D_O is rather constant because of the stable internal fabric and the minimization of friction development at particle contacts (Johnston 1981; Santamarina et al. 2001).

TABLE 1—*Grain-size characteristics, compaction and shear strength parameters of materials used.*

Laboratory material	Parent soil[a]	D_{50} (mm)[b]	C_u[c]	C_c[d]	USCS[e]	$\gamma_{d,max}$ (kN/m^3)[f]	$\gamma_{d,min}$ (kN/m^3)[g]	e_{min}[h]	e_{max}[i]	ϕ' (deg.)[j]
N1	Natural sand	0.27	1.58	0.93	SP	16.3	13.1	0.608	1.008	41.3
N2	Natural sand	0.56	2.76	1.23	SP	17.8	14.2	0.467	0.841	40.6
N3	Natural sand	0.60	1.34	0.84	SP	16.1	13.3	0.628	0.963	—
N4	Natural sand	1.33	2.13	1.01	SP	—	—	—	—	—
Q1	Quarry sand	0.16	2.00	0.99	SP	—	—	—	—	46.7
Q2	Quarry sand	0.85	3.23	0.78	SP	—	—	—	—	—
Q3	Quarry sand	1.33	2.13	1.01	SP	—	—	—	—	—
Q4	Quarry sand	2.00	2.50	1.07	SP	—	—	—	—	—
V1	Volcanic sand	0.23	1.53	0.73	SP	13.3	10.2	0.740	1.269	44.0
V2	Volcanic sand	0.53	2.23	1.28	SP	—	—	—	—	—
V3	Volcanic sand	0.55	4.18	0.75	SP	14.6	11.5	0.590	1.017	43.2
V4	Volcanicsand	0.92	2.00	0.64	SP	—	—	—	—	—
V5	Volcanic sand	1.60	2.12	0.94	SP	—	—	—	—	—

[a]Natural sand: River sand of sub-rounded to rounded quartz particles ($G_s = 2.67$). Quarry sand: Crushed rock of sub-angular to angular quartz particles ($G_s = 2.67$). Volcanic sand: Crushed volcanic rhyolithic glassy rock of sub-angular to angular porous particles ($G_s = 2.36$).
[b]Mean grain size.
[c]Coefficient of uniformity $C_u = D_{60}/D_{10}$.
[d]Coefficient of curvature $C_c = D_{30}^2/(D_{60} \times D_{10})$.
[e]ASTM D2487-00 (2000).
[f,h]ASTM D1557-02 (2002).
[g,i]ASTM D4254-00 (2000).
[j]Angle of maximum shear strength, ASTM D3080-03 (2003).

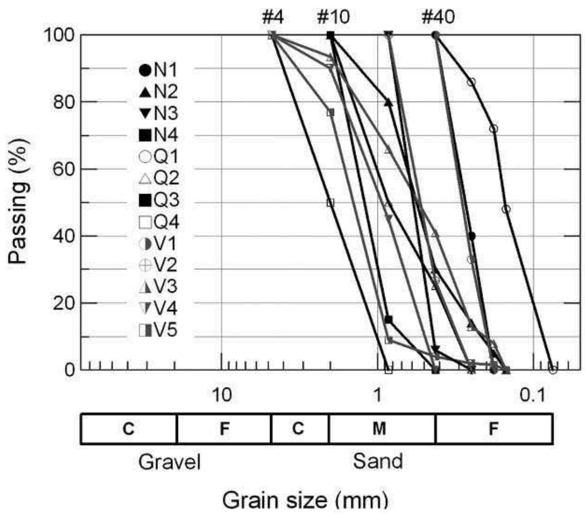

FIG. 1—*Grain-size distribution curves of materials used (note: materials N4 and Q3 exhibit the same grain-size distribution curve).*

Most of the researches referred to above have been focused on the dynamic properties of granular soils predominately composed of quartz particles. A specific category of granular soils concerns volcanic sands, often found in crushed form (artificially crushed rock), composed of rhyolitic glassy rock. Because of the geologic and physical procedures of the formation of the parent rock, the aforementioned volcanic soils exhibit intra-particle voids, which results in remarkably lower bulk density and higher void ratio of these materials in comparison to the corresponding values that the quartz soils exhibit. According to this, the aforementioned soils comprise promising lightweight geo-materials potentially used in civil engineering applications specifically in structures and geo-structures for which the reduction of the vertical and/or lateral pressures is compulsory as for example as construction material in high embankments overlying soft-compressible soils or as fill material in retaining walls.

The present paper partially contributes to fill the literature gap relative to the dynamic properties of volcanic granular soils. Specifically, we present a synthesis of low-amplitude torsional resonant column test results on reconstituted dry sands composed of rhyolitic glassy rock (volcanic artificially crushed rock) with additional test results on quartz granular soils composed of natural and quarry-crushed sands. All experiments were performed in the Laboratory of Soil Mechanics, Foundations and Geotechnical Earthquake Engineering of Aristotle University in Thessaloniki, Greece.

Materials and Methodology

Materials Used

In this paper, 13 uniform to poor graded sands classified as SP according to ASTM D2487-00 (USCS) specification were studied. The grain-size characteristics of the materials tested are summarized in Table 1 and Fig. 1. Materials N1, N2, N3, and N4 are composed of natural (river) sand, whereas materials Q1, Q2, Q3, and Q4 are composed of quarry sandy gravel (artificially crushed rock). The aforementioned soils are predominately composed of quartz particles of specific gravity, G_s, equal to 2.67. Additionally, five materials namely V1, V2, V3, V4, and V5 were included in this study. These soils are composed of rhyolitic glassy rock

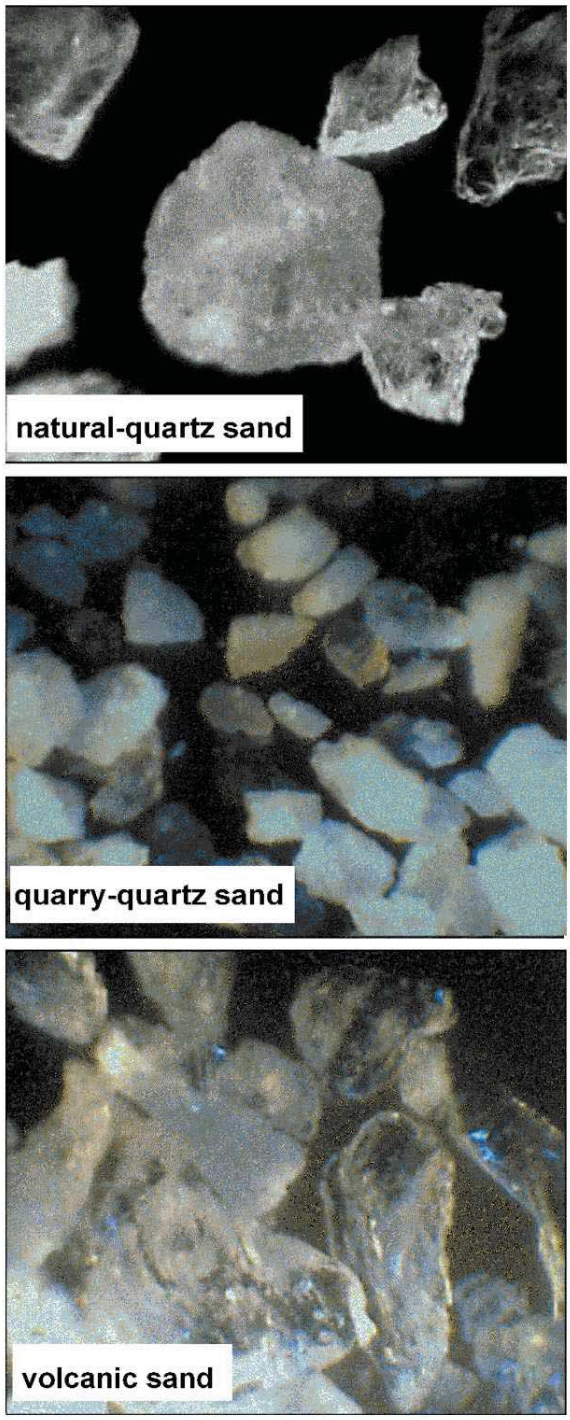

FIG. 2—*Images of individual particles of representative samples of this study (natural sand: N2, quarry sand: Q2, volcanic sand: V1).*

TABLE 2—*Torsional resonant column testing program.*

Code name of specimen	Laboratory material	γ_{do} (kN/m^3)	e_o	D_{ro} (%)	γ_{LA} (%)[a]
N1-1	N1	15.8	0.661	87	$4.8 \times 10^{-4} - 6.4 \times 10^{-4}$
N1-2	N1	13.3	0.964	11	$4.9 \times 10^{-4} - 1.1 \times 10^{-3}$
N2-1	N2	17.3	0.517	87	$5.6 \times 10^{-4} - 8.1 \times 10^{-4}$
N2-2	N2	16.6	0.577	71	$2.7 \times 10^{-4} - 5.9 \times 10^{-4}$
N2-3	N2	16.5	0.588	68	$4.8 \times 10^{-4} - 8.2 \times 10^{-4}$
N2-4	N2	15.6	0.682	43	$6.1 \times 10^{-4} - 8.8 \times 10^{-4}$
N3-1	N3	15.6	0.680	85	$7.6 \times 10^{-4} - 1.1 \times 10^{-3}$
N3-2	N3	13.5	0.938	8	$8.9 \times 10^{-4} - 1.1 \times 10^{-3}$
N4-1	N4	15.5	0.566	—	$2.7 \times 10^{-4} - 5.7 \times 10^{-4}$
N4-2	N4	16.7	0.571	—	$3.5 \times 10^{-4} - 1.3 \times 10^{-3}$
N4-3	N4	16.5	0.588	—	$3.9 \times 10^{-4} - 5.4 \times 10^{-4}$
Q1-1	Q1	15.6	0.683	—	$4.2 \times 10^{-4} - 6.2 \times 10^{-4}$
Q1-2	Q1	13.4	0.954	—	$1.1 \times 10^{-3} - 1.1 \times 10^{-3}$
Q2-1	Q2	17.0	0.545	—	$2.9 \times 10^{-4} - 4.1 \times 10^{-4}$
Q2-2	Q2	15.0	0.742	—	$6.0 \times 10^{-4} - 7.5 \times 10^{-4}$
Q3-1	Q3	16.4	0.594	—	$3.4 \times 10^{-4} - 6.6 \times 10^{-4}$
Q3-2	Q3	14.4	0.820	—	$6.2 \times 10^{-4} - 9.6 \times 10^{-4}$
Q4-1	Q4	16.9	0.553	—	$2.0 \times 10^{-4} - 3.7 \times 10^{-4}$
Q4-2	Q4	14.8	0.770	—	$6.3 \times 10^{-4} - 7.0 \times 10^{-4}$
V1-1	V1	12.7	0.823	84	$7.1 \times 10^{-4} - 8.6 \times 10^{-4}$
V1-2	V1	11.1	1.083	44	$1.0 \times 10^{-3} - 1.6 \times 10^{-3}$
V2-1	V2	11.8	0.961	—	$6.4 \times 10^{-4} - 1.9 \times 10^{-3}$
V2-2	V2	10.3	1.250	—	$1.7 \times 10^{-3} - 2.1 \times 10^{-3}$
V3-1	V3	12.9	0.794	52	$5.8 \times 10^{-4} - 8.0 \times 10^{-4}$
V3-2	V3	12.2	0.898	28	$1.1 \times 10^{-3} - 1.5 \times 10^{-3}$
V3-3	V3	11.6	0.991	6	$1.1 \times 10^{-4} - 1.7 \times 10^{-4}$
V4-1	V4	11.9	0.943	—	$2.8 \times 10^{-4} - 1.1 \times 10^{-3}$
V4-2	V4	10.6	1.194	—	$2.2 \times 10^{-3} - 2.5 \times 10^{-3}$
V5-1	V5	11.7	0.976	—	$5.3 \times 10^{-4} - 8.9 \times 10^{-4}$
V5-2	V5	11.5	1.011	—	$9.8 \times 10^{-4} - 1.5 \times 10^{-3}$
V5-3	V5	10.4	1.229	—	$1.9 \times 10^{-3} - 2.3 \times 10^{-3}$

Note: Low-amplitude tests were performed at increasing steps of σ'_m equal to 25, 50, 100, and 200 kPa.
[a]Shearing strain amplitude where G_O and DT_O are defined.

(volcanic artificially crushed rock) with specific gravity, G_s, equal to 2.36. G_s values of the materials under study were determined according to ASTM D854-02 (2002) specification. It is noticed that the quartz sands of this study are standard materials tested at the Laboratory of Soil Mechanics, Foundations and Geotechnical Earthquake Engineering of Aristotle University in Thessaloniki, Greece (e.g., Anastasiadis 1994; Pitilakis and Anastasiadis 1994; Senetakis 2011; Senetakis et al. 2012a, 2012b).

According to literature-empiric diagrams (Krumbein and Sloss 1963; after Santamarina et al. 2001) and after observing the individual particles of representative samples through a microscope, the natural-quartz sands comprise sub-rounded to rounded particles, whereas the quarry-quartz and the volcanic sands comprise sub-angular to angular particles. It is noticed at this point in the literature, that more sophisticated and rigorous methods for the quantitative estimation of particle roundness have been proposed [summarized by Blott and Pye (2008) and Cavarretta (2009)], whereas it is evident that the overall response of granular soils under both monotonic and cyclic loading is also affected by the sphericity of the particles, as well as the surface roughness (e.g., Santamarina and Cascante 1998; Clayton et al. 2004; Cho et al. 2006; Bui et al. 2007; Cavarretta 2009; Cavarretta et al. 2010). Moreover, in a recent study by Cho et al. (2006) a new parameter, namely the regularity, was introduced to correlate the response of granular materials with the shape of particles in terms of both roundness and sphericity. However, on the framework of this paper, we simply focus on the macro-scale behavior of the sands under study, where the effect of the shape of particles on the overall response of the specimens is qualitatively considered in terms of mean roundness. Specifically, the materials under study are distinguished in two general categories through optical view of the individual particles; natural sands of sub-rounded to rounded particles and crushed sands of sub-angular to angular particles. Images of the individual particles of representative samples are given in Fig. 2.

The compaction characteristics in terms of minimum and maximum void ratio and dry unit weight as well as the shear strength in terms of the angle of shearing resistance (φ') of representative

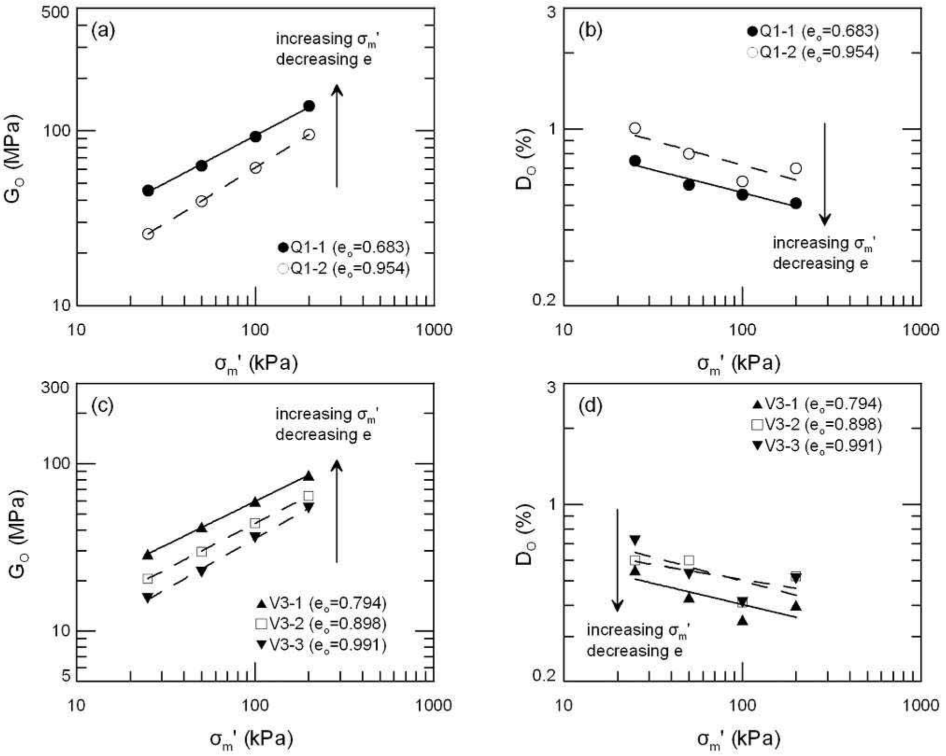

FIG. 3—*Representative low-amplitude resonant column test results: Effect of mean effective confining pressure and void ratio on the small-strain shear modulus and damping ratio.*

samples are also presented in Table 1. The angle of shearing resistance was determined in a direct shear apparatus on dry specimens prepared at high relative density of the solid matrix. Direct shear tests were performed at variable normal stresses ($\sigma_n{}' = 35$, 67, and 99 or 131 kPa) and the resultant values of φ' correspond to the maximum shear strength of the samples. Details of the compaction testing program and the direct shear tests of the materials under study are given in Senetakis (2011). It is observed that the natural sands exhibit "typical" values of $\gamma_{d,\min}$ and $\gamma_{d,\max}$ on the order of 13.0 to 14.0 kN/m^3 and 16.0 to 18.0 kN/m^3, respectively, depending on the grain-size distribution, whereas the volcanic sands exhibit systematically lower values of $\gamma_{d,\max}$ and $\gamma_{d,\min}$ in comparison to the quartz ones. This is mainly attributed to the intra-particle voids of the volcanics soils and to the variability in particles density between quartz and volcanic sands. Additionally, the volcanic sands exhibit high shear strength with φ' values comparable to the respective values of the quartz granular soils. The φ' values of the volcanic sands under study are comparable to the corresponding values presented by Orense et al. (2006) on reconstituted volcanic sands.

Experimental Equipment, Specimens Preparation, and Testing Program

Cyclic tests were performed on reconstituted dry specimens of variable relative density (or void ratio) values using a longitudinal-torsional resonant column (RC) apparatus that follows the fixed-free configuration (Drnevich 1967). Details of the

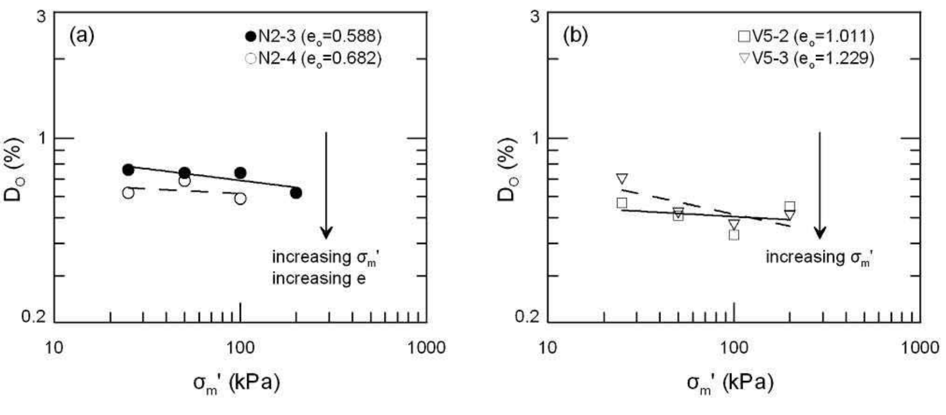

FIG. 4—*Representative low-amplitude resonant column test results: Effect of mean effective confining pressure and void ratio on the small-strain damping ratio.*

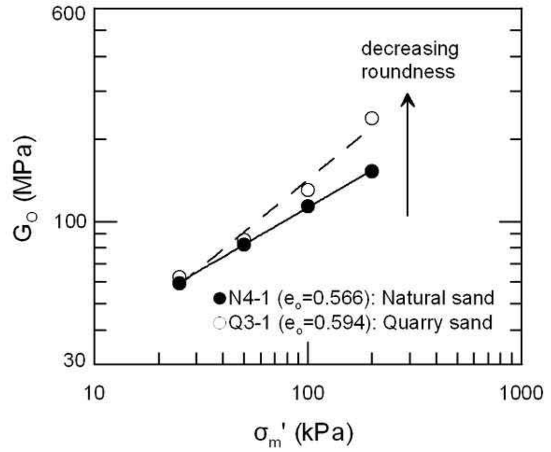

FIG. 5—*Effect of particles shape on the small-strain shear modulus of quartz sands of similar grain-size distribution (specimens N4-1 and Q3-1 were constructed at the same compaction energy).*

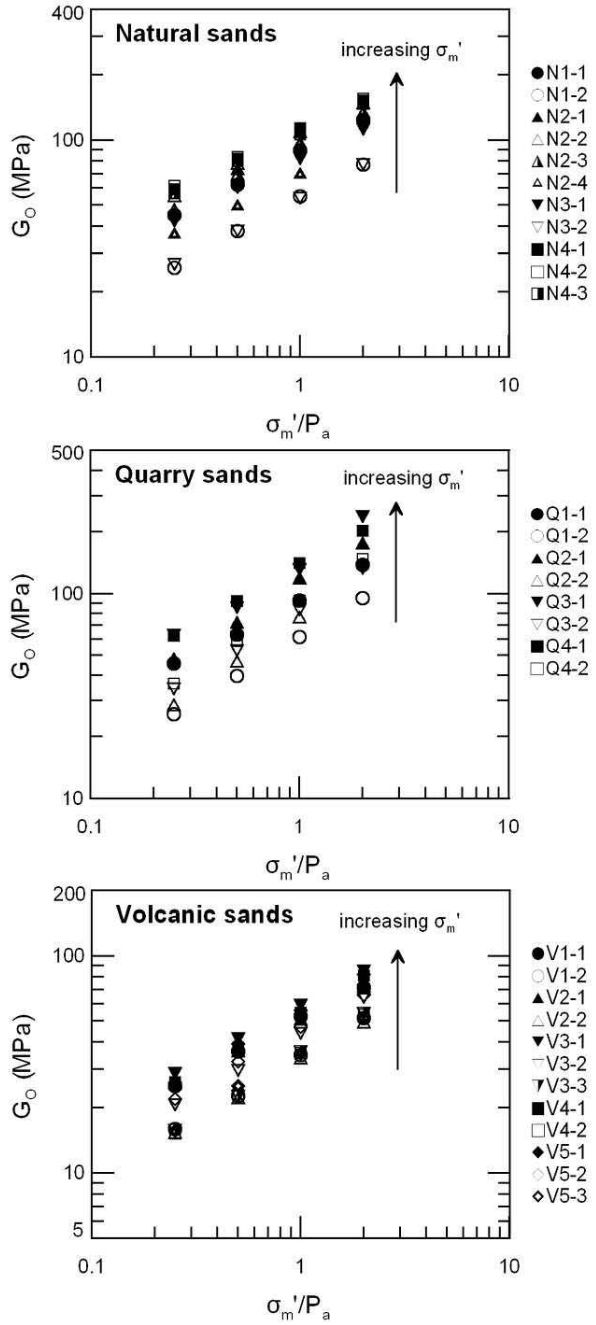

FIG. 6—*Synopsis of the experimental small-strain shear modulus values against the mean effective confining pressure of all specimens.*

experimental equipment used are given in Senetakis (2011). The bottom (passive) end of the specimen is rigidly fixed on a base pedestal, whereas the sinusoidal excitation is applied at the top (active) end of the specimen. For this purpose, two magnets are attached on the excitation mechanism, which is fixed on the top end of the specimen and four coils surround the magnets. By introducing current in the coils, an electromagnetic field is produced that imposes the magnets, and therefore the top end of the specimen, in sinusoidal excitation. The experimental procedure is manually controlled and monitored through electronic equipment. The axial deformations of the specimen are obtained through a linearly variable differential transformer (LVDT), whereas for the determination of the radial deformations during the elevation of the mean confining pressure we assume isotropic compression of the specimen. For saturated samples, the volume change of the specimen is obtained by a GDS controller/regulator. The torsional resonant column tests and the analysis of the results were performed according to the ASTM D4015-92 (1992) specifications.

Table 2 summarizes the torsional resonant column testing program as well as details of the specimens under study (initial dry density, γ_{do}, initial void ratio, e_o, initial relative density, D_{ro}, and the shearing strain amplitude, γ_{LA}, where G_O and D_O are defined). Thirty-one dry specimens of approximately 71 mm in diameter and 142 mm in height were tested; eleven specimens of natural sands of quartz particles, eight specimens of quarry-crushed sands of quartz particles, and twelve specimens of volcanic sands. To examine the effect of void ratio on the dynamic response of the samples, specimens of the same material were constructed at variable compaction energy; loose specimens were constructed by hand-spooning of the material into a metal mold in the RC device, whereas medium dense and dense to very dense specimens were constructed in many layers with increasing compaction tips to the top using a small metal rod of diameter approximately half the diameter of the specimen.

Low-amplitude resonant column tests were performed at increasing mean effective confining pressure steps, 25, 50, 100, and 200 kPa. The effects of the mean effective confining pressure (σ_m'), the void ratio (e), the grain-size characteristics (in terms of mean grain size, D_{50}, and coefficient of uniformity, C_u) the particles shape (by distinguishing the soils in sands of sub-rounded to rounded particles and sands of sub-angular to angular particles), as well as the effect of particle morphology and mineralogy (by distinguishing the soils in sands of quartz particles and volcanic sands composed of rhyolitic glassy rock) on the small-strain shear modulus (G_O) and the small-strain damping ratio (D_O) are studied.

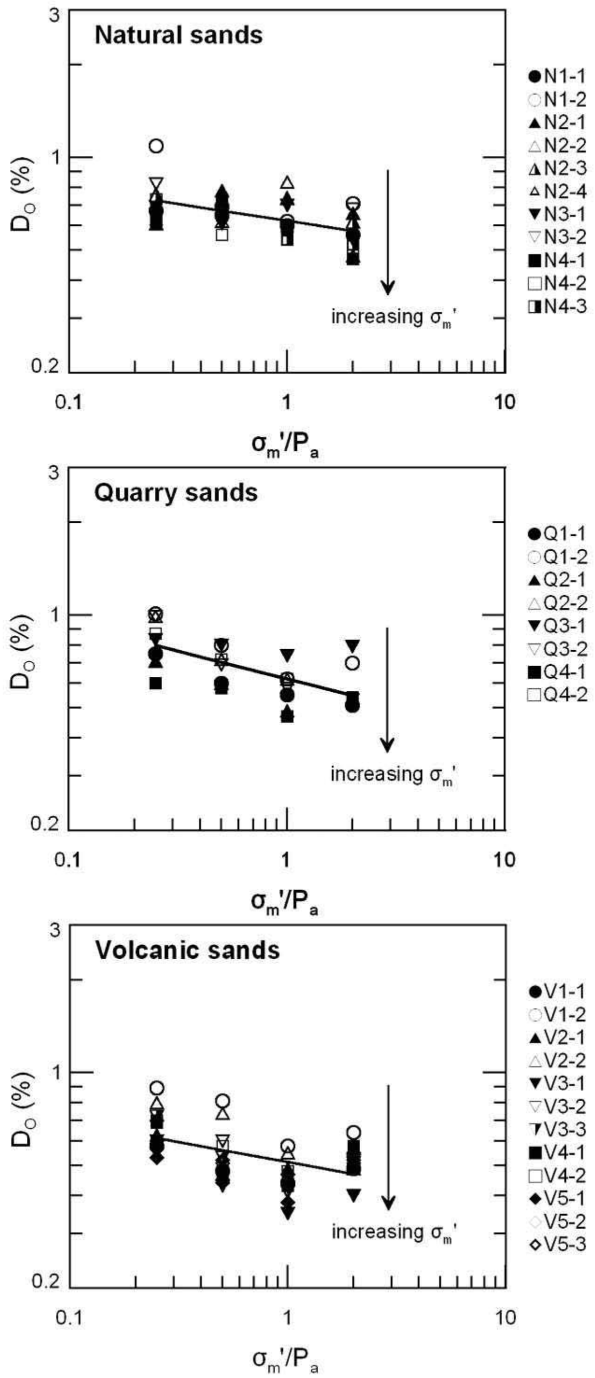

FIG. 7—*Synopsis of the experimental small-strain damping ratio values against the mean effective confining pressure of all specimens.*

Experimental Results and Discussion

Synopsis of Experimental Results

The effect of the mean effective confining pressure (σ_m') and the void ratio (e) on the small-strain shear modulus (G_O) and damping ratio (D_O) of representative samples is illustrated in Figs. 3 and 4. As expected, G_O is strongly affected by σ_m' and e; G_O values of the quarry-crushed sand Q1 (Fig. 3(*a*)) and the volcanic sand V3 (Fig. 3(*c*)) increase with increasing σ_m' and decreasing e. As depicted in Figs. 3(*b*) and 3(*d*), D_O values slightly decrease with increasing σ_m' and decreasing e. However, there was not observed a monotonic trend of the effect of void ratio on D_O of all specimens. As shown, for example, in Fig. 4(*a*), at a given σ_m', D_O of the natural sand N2 decreases with increasing void ratio, whereas in Fig. 4(*b*) there is not observed a clear trend of the effect of void ratio on the small-strain damping ratio of the volcanic sand V5. Similar findings were reported by Menq (2003) on dry coarse grained soils. It seems therefore that the laboratory investigations do not show a strong effect of void ratio on the small-strain damping ratio. The latter remark is true at least for dry granular soils for which the available literature data (e.g., Menq 2003; Zambelli et al. 2006) indicate that the effect of the loading frequency on the small-strain damping ratio is not that important.

Figure 5 illustrates a representative example of the effect of particles shape on the small-strain shear modulus of the quartz sands. In this figure, the G_O values of two sands of identical grain-size distribution curves are plotted against the mean effective confining pressure. Material N4 corresponds to a natural sand of sub-rounded to rounded quartz particles, and material Q3 corresponds to a quarry sand of sub-angular to angular quartz particles. The specimens of this figure were prepared at the same compaction energy (same number of layers and compaction tips) at high relative density of the solid matrix. In the range of σ_m' values at which specimens were tested (25 to 200 kPa), the quarry sand Q3 exhibits systematically higher G_O values in comparison to the natural sand N4. Assuming linear increase of the small-strain shear modulus with increasing σ_m' in log-scale plot, the slope of the diagram G_O-σ_m' is steeper in the case of the quarry sand. According to Cho et al. (2006), angular particles exhibit in general higher deformability at particle-to-particle interfaces and thus, the increase of the confining pressure leads to more pronounced increase of particle contacts and consequently to higher rate of G_O increase. The higher G_O values of the quarry sand Q3 in comparison to the natural sand N4 in the range of σ_m' under study are attributed to the steeper slope of the diagram G_O-σ_m'. If we extended the fitting curves of Fig. 5 to lower σ_m' values, in the region of 1 to 25 kPa, we would observe that the natural sand exhibits higher G_O values in comparison to the quarry one. The latter remark is partially attributed to the looser packing that soils of angular particles exhibit in comparison to soils of rounded particles, which is true for low confining pressures where the developed normal forces at particle contacts are not of significant magnitude (Youd 1973; Santamarina et al. 2001; Cho et al. 2006).

In Figs. 6 and 7, the experimental low-amplitude resonant column test data of this study in terms of small-strain shear modulus (G_O) and small-strain damping ratio (D_O) against the mean effective confining pressure expressed as σ_m'/P_a of all specimens are presented. Specimens are distinguished in these figures in three groups: natural quartz sands, quarry quartz sands, and volcanic sands. G_O values of all specimens systematically increase whereas D_O values slightly decrease with increasing σ_m'. At a given value of the normalized confining pressure σ_m'/P_a, the variability of the G_O values in each group of sands is mainly attributed to the variability of void ratio (e) and coefficient of uniformity (C_u). Regarding the D_O values, there was not observed a monotonic trend of

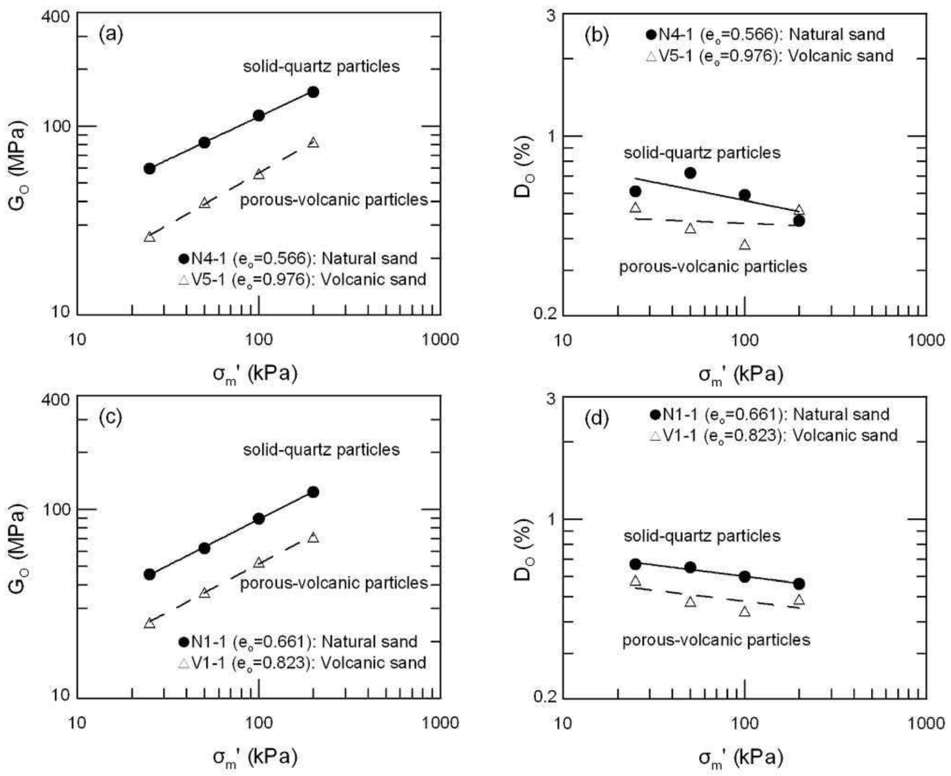

FIG. 8—*Comparison of small-strain shear modulus and damping ratio values between quartz and volcanic sands of similar grain-size distribution.*

the effect of e or C_u, and thus the variability of the D_O values at a given normalized confining pressure σ'_m/P_a is not explained as in Fig. 6. In agreement with the literature data (e.g., Laird 1994; Menq 2003) and concerning the range of σ'_m of this study (25 to 200 kPa), all dry sands exhibit D_O values lower than unity.

In Fig. 8, the G_O and D_O values of quartz sands of solid particles and volcanic sands of porous particles of similar grain-size distribution are compared. Specimens N4-1 and V5-1 of Figs. 8(*a*) and 8(*b*) as well as specimens N1-1 and V1-1 of Figs. 8(*c*) and 8(*d*) were prepared at the same compaction energy. It is noticed that the volcanic sands exhibit significantly lower values of G_O in comparison to the quartz ones; this is mainly attributed to the variability in particles density, mineralogy, and morphology between volcanic and quartz sands as well as the overall higher void ratio and lower dry density that the volcanic sands exhibit in comparison to the quartz ones. It is also noticed that the volcanic sands exhibit slightly lower values of D_O in comparison to the quartz ones, which is possibly explained by the variability in particles mineralogy and morphology.

Because the mechanical characteristics of the volcanic particles are not well known, further research is needed on this topic, including for example macro-scale tests where the overall response of the granular assembly at elevated stresses will be evaluated (e.g., one-dimensional compression tests) as well as micro-scale tests where the deformability at particle contacts, the normal and shear stiffness at particle contacts as well as the strength of the individual particles will be examined. The aforementioned tests would provide invaluably an insight in the mechanical and dynamic response of the volcanic sands as well as the key parameters that are necessary as inputs for further research through numerical modeling using the discrete element method (Cundall and Strack 1979) of assemblies of quartz and volcanic sands under monotonic and cyclic loading. It is believed that a thorough investigation through numerical modeling could invaluably contribute to precisely explain the main mechanisms that dominate at the volcanic assemblies, as well as the differences in the observed responses between quartz and volcanic sands under cyclic loading.

Comparison Between Experimental Results and Literature Data

In Fig. 9, we plot the measured G_O values of this study against the estimated values derived from empirical relationships proposed in the literature for granular soils. For this purpose, we use herein the well-known expressions for the estimation of G_O proposed by Hardin and Richart (1963), Hardin (1978), Saxena and Reddy (1989), Menq (2003), and Wichtmann and Triantafyllidis (2009). Hardin and Richart (1963) proposed separate relationships for sands of rounded and sands of angular particles and thus, we use herein the aforementioned relationships separately for the natural-quartz sands of sub-rounded to rounded particles (Fig. 9(*a*)) and for the quarry-quartz and the volcanic sands of sub-angular to angular particles (Figs. 9(*b*) and 9(*c*), respectively). In addition, Menq (2003) proposed an empirical relationship for the estimation of G_O that includes the parameters C_u and D_{50}, derived from tests on natural granular soils, whereas Wichtmann and Triantafyllidis (2009) proposed a similar relationship that includes the parameter C_u, derived from tests on coarse soils of sub-angular particles.

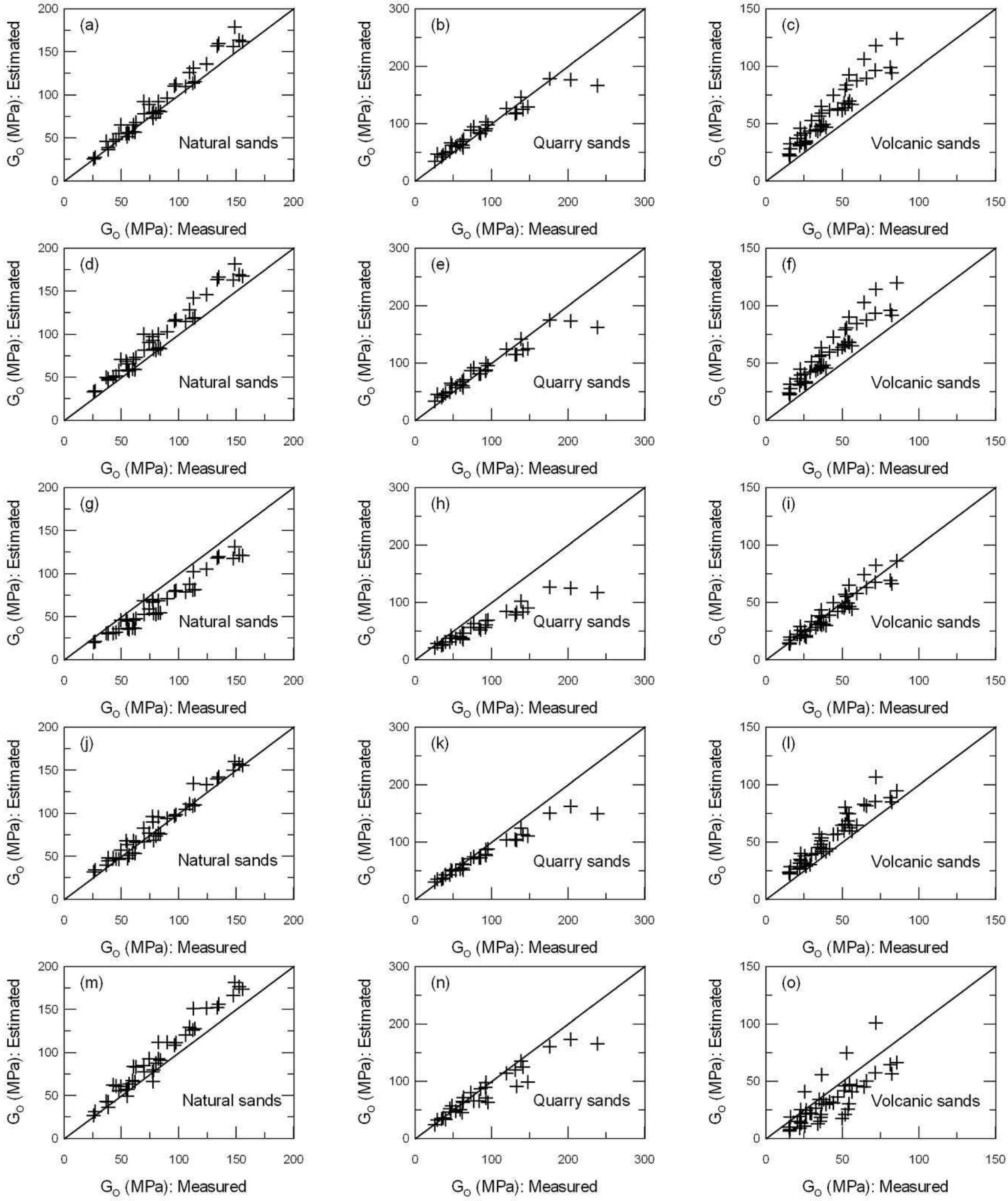

FIG. 9—*Measured small-strain shear modulus values of this study against estimated values resultant from empirical relationships proposed by: (a)–(c) Hardin and Richart (1963), (d)–(f) Hardin (1978), (g)–(i) Saxena and Reddy (1989), (j)–(l) Menq (2003), and (m)–(o) Wichtmann and Triantafyllidis (2009).*

Except for the relationship proposed by Saxena and Reddy (1989), most empirical relationships predict satisfactorily or slightly over- or underestimate the G_O values of the quartz sands. For the natural sands, the equation proposed by Menq (2003) seems to be more valid, which is possibly explained by the similar mineralogy and particle shape between the sands tested in this study and in the experimental investigation by Menq (Fig. 9(*j*)). The empirical relationship proposed by Wichtmann and Triantafyllidis (2009) slightly underestimates the G_O values of the quarry sands (Fig. 9(*n*)), specifically for measured G_O values on the order

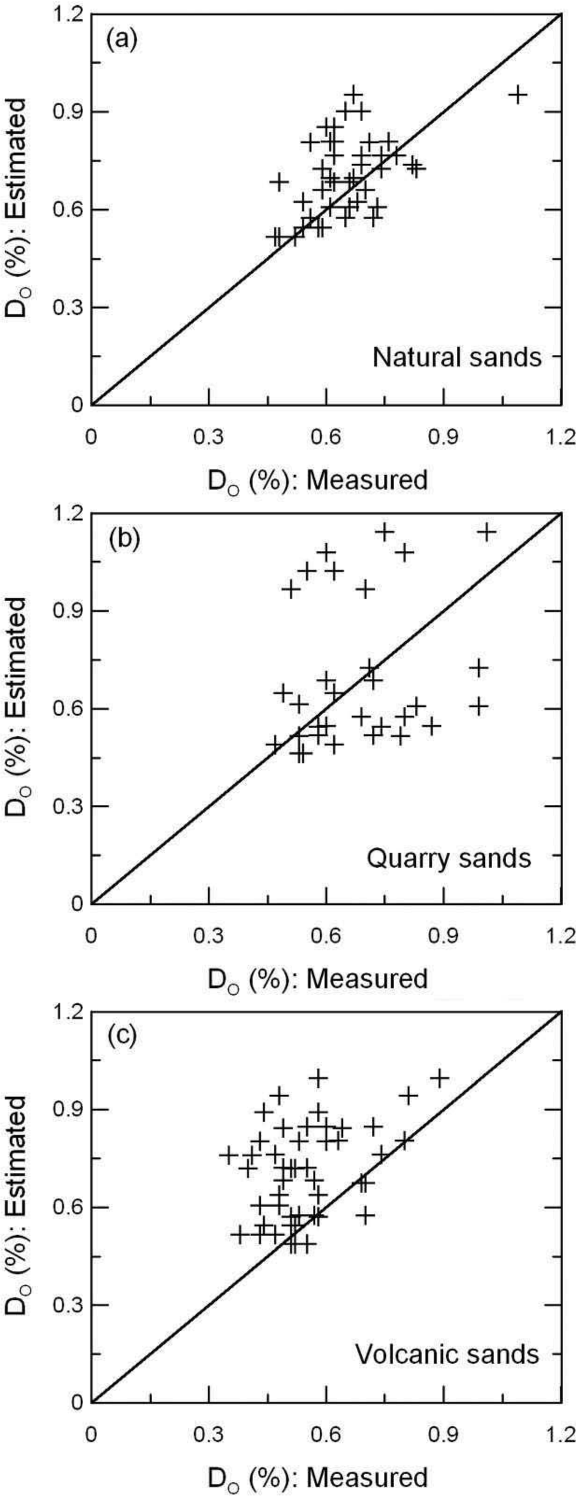

FIG. 10—*Measured small-strain damping ratio values of this study against estimated values resultant from empirical relationships proposed by Menq (2003).*

of 100 to 250 MPa and thus, for mean effective confining pressures beyond 50 kPa. These differences are possibly attributed to the variability in particles roundness between the sands included in this study and the ones tested by Wichtmann and Triantafyllidis (2009).

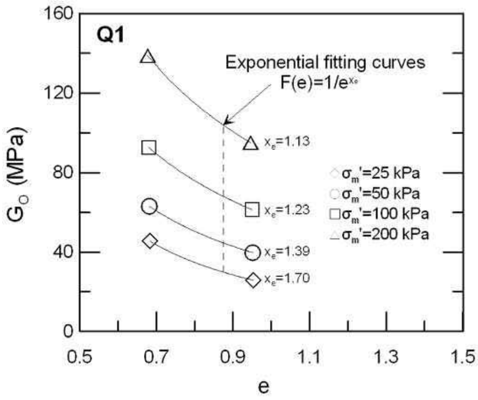

FIG. 11—*Example of estimating the exponent x_e of the sands under study through regression analysis (Q1: quarry fine-grained sand).*

The empirical equation proposed by Saxena and Reddy (1989) systematically underestimates the G_O values of the quartz sands (Figs. 9(*g*) and 9(*h*)). This is attributed to the experimental equipment that Saxena and Reddy (1989) used in their investigation. According to to Saxena and Reddy, the modified Drnevich RC device used in their study systematically underestimates the shear stiffness of sandy soils. Surprisingly, the latter equation, derived from tests on quartz sands, was the only one, out of many available equations in the literature that the authors examined, that estimated satisfactorily the shear stiffness of the volcanic sands (Fig. 9(*i*)).

In Fig. 10, the measured against the analytically derived D_O values of the sands under study are compared. For this purpose, we use the empirical relationship proposed by Menq (2003) in which the small-strain damping ratio is a function of mean grain size, coefficient of uniformity and mean effective confining pressure. For practical purposes, the relationship proposed by Menq (2003) estimates satisfactorily the D_O values of the natural sands, whereas systematically overestimates the D_O values of the

TABLE 3—*Exponent* x_e *of void ratio function of all specimens tested.*

	x_e			
Code name of material	σ'_m= 25 kPa	σ'_m= 50 kPa	σ'_m= 100 kPa	σ'_m= 200 kPa
N1	1.48	1.30	0.40[a]	0.33[a]
N2	2.17	1.58	1.69	0.87
N3	1.43	1.43	1.31	1.19
N4	1.47	0.88	1.79	1.04
Q1	1.70	1.39	1.23	1.13
Q2	1.68	1.43	1.41	—
Q3	1.86	1.51	1.39	1.79
Q4	1.62	1.37	1.27	0.98
V1	1.65	1.71	1.52	1.20
V2	1.99	1.88	1.56	1.38
V3	2.72	2.77	2.25	2.07
V4	2.29	2.09	1.79	1.76
V5	2.05	1.72	1.82	1.75

[a]Not included in the regression analysis.

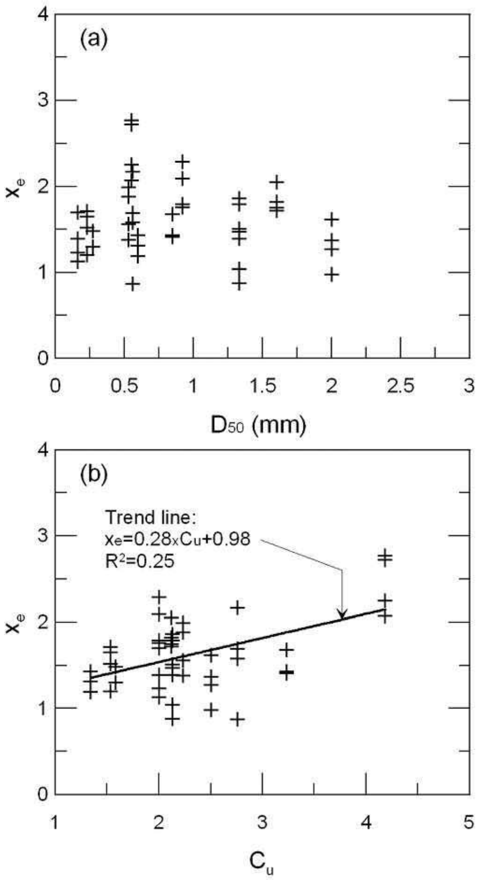

FIG. 12—*Effect of (a) mean grain size, and (b) coefficient of uniformity on the exponent* x_e.

volcanic sands. The latter remark is partially attributed to the variability in mineralogy and surface roughness between the soils tested by Menq (2003) and the volcanic sands of this study. However, in both this investigation, as well as in Menq (2003), the surface roughness of the coarse particles was not quantitatively estimated. Additionally, the variability in the elastic properties and mineralogy of the particles plays an important role but as will be stated later in this paper, further research is needed on this topic. Overall, the variability between experimental and analytically derived values of Fig. 10 includes the additional uncertainties on estimating the small-strain damping ratio in the laboratory, which uncertainties introduce some scatter on the data of Figs. 10(*a*) to 10(*c*).

General Form of Empirical Relationships Used—Proposed Void Ratio Function

The small-strain shear modulus and damping ratio of the sands under study are correlated to the mean effective confining pressure (σ'_m) through the general forms of Eqs 3 and 4, respectively. In these equations, A_G and A_D are parameters of the regression analysis that depend on particles shape, distribution and mineralogy, $F(e)$ is the void ratio function and the exponents n_G and n_D express the slope of the G_O-σ'_m and D_O-σ'_m diagrams in log scale, respectively. The general forms of Eqs 3 and 4 were also used by Menq (2003) who studied the dynamic properties of sandy and gravelly specimens in a free-free resonant column apparatus.

$$G_O = A_G \times F(e) \times \left(\frac{\sigma'_m}{P_a}\right)^{n_G} \tag{3}$$

$$D_O = A_D \times \left(\frac{\sigma'_m}{P_a}\right)^{n_D} \tag{4}$$

The void ratio function adopted herein follows the general form of Eq 5. This exponential $F(e)$ function was also used by Jamiolkowski et al. (1991) and Menq (2003). Jamiolkowski et al. (1991) proposed a value of the exponent x_e equal to 1.3 for sands, whereas Menq (2003) proposed an equation that correlates the exponent x_e with the mean grain size (D_{50}) of sands and gravels. Specifically, according to the equation proposed by Menq (2003), the exponent x_e increases from 1.11 to 1.35 as D_{50} increases from 1.0 to 5.0 mm.

$$F(e) = \frac{1}{e^{x_e}} \tag{5}$$

In Fig. 11, a simple example of determining the exponent x_e of Eq 5 is illustrated. In specific, we plot the G_O against the void ratio values of all specimens of the same material prepared at variable initial relative density values and then we determine, through regression analysis, the exponent x_e at elevated mean effective confining pressures. This was done for all the 31 specimens of this study. In Table 3, the values of the exponent x_e of all specimens are summarized, whereas in Fig. 12 the x_e values are plotted against the mean grain size, D_{50} (Fig. 12(*a*)), and the coefficient of uniformity, C_u (Fig. 12(*b*)). The sands of this study exhibit a mean value of x_e equal to 1.62 with a standard deviation equal to 0.41. As shown in Fig. 12, x_e slightly increases with increasing C_u, whereas the overall effect of D_{50} on the exponent x_e is not clear. Even though the overall effect of C_u on the exponent x_e is not that strong because the correlation coefficient, R^2, is equal to 0.25 as shown in Fig. 12(*b*), we decided to analytically express the exponent x_e as a function of C_u and thus, Eq 5 is modified to Eq 6. Consequently, the determination of the parameters A_G and n_G of all specimens is implemented through Eq 7.

$$F(e) = \frac{1}{e^{0.28 \times C_u + 0.98}} \tag{6}$$

$$G_O \times e^{0.28 \times C_u + 0.98} = A_G \times \left(\frac{\sigma'_m}{P_a}\right)^{n_G} \tag{7}$$

Analysis of Experimental Data—Proposed Empirical Relationships

In Figs. 13 and 14, we plot separately for the natural quartz sands, the quarry quartz sands and the volcanic granular soils the parameters A_G, A_D, n_G, and n_D stemming from the regression analysis of the experimental data assuming linear increase of the small-strain shear modulus, G_O, and linear decrease of the small-strain damping ratio, D_O, with increasing σ'_m in log-scale plots. The aforementioned parameters, summarized also in Table 4, are plotted against the coefficient of uniformity, C_u. In Figs. 13(*a*), 13(*c*), and 13(*e*), it is observed that A_G values decrease with increasing C_u, which is

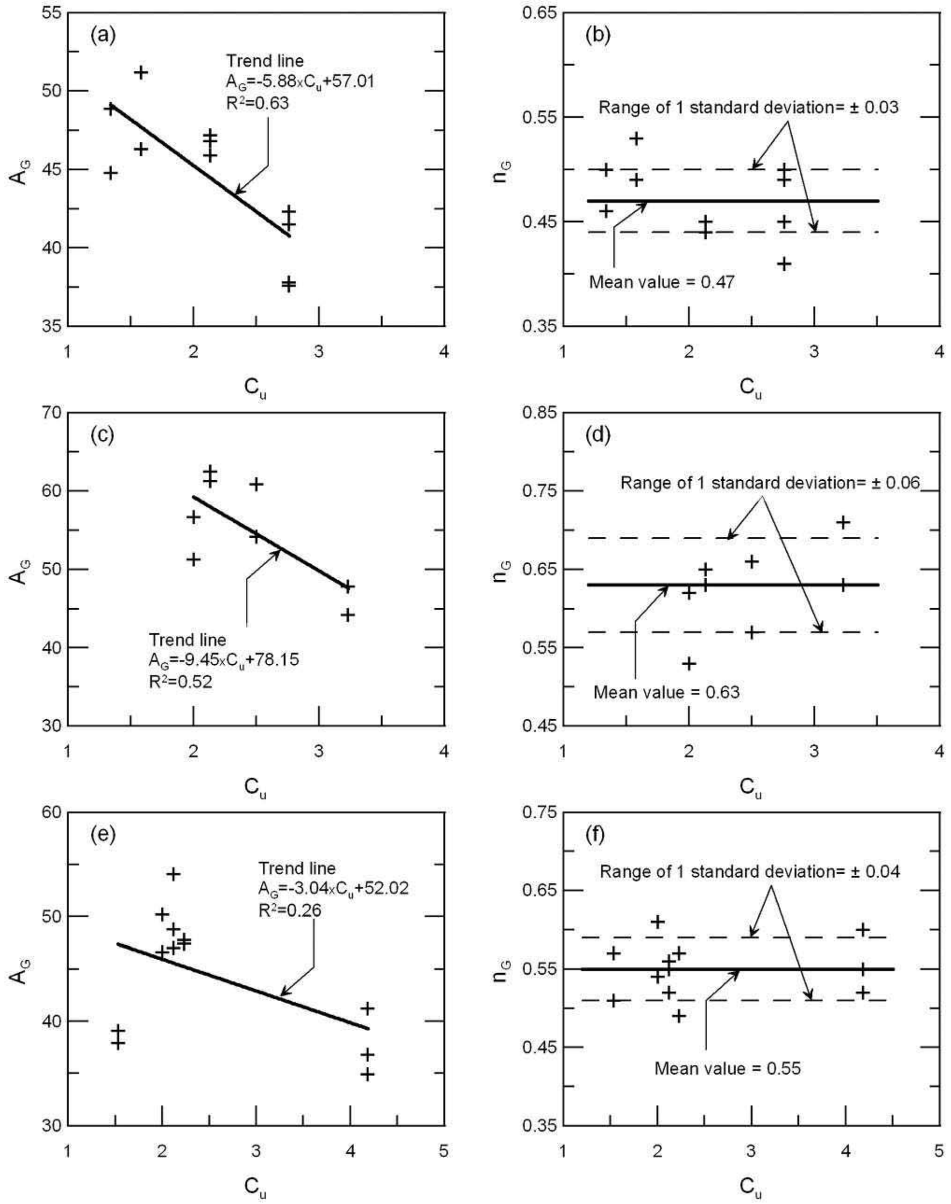

FIG. 13—*Effect of the coefficient of uniformity, C_u, on the parameter A_G and the exponent n_G of (a)–(b) natural-quartz sands, (c)–(d) quarry-quartz sands, and (e)–(f) volcanic sands.*

consistent with the findings by Menq (2003). Consequently, we correlate herein the parameter A_G with the coefficient of uniformity, C_u, through Eqs 8(*a*), 8(*b*), and 8(*c*) separately for the natural quartz sands, the quarry quartz sands, and the volcanic granular soils, respectively. However, it is noticed that the overall correlation between A_G and C_u with respect to the volcanic sands is not that strong as in the quartz soils.

$$\text{Natural quartz sands}: A_G = -5.88 \times C_u + 57.01 \tag{8a}$$

$$\text{Quarry quartz sands}: A_G = -9.45 \times C_u + 78.15 \tag{8b}$$

$$\text{Volcanic sands}: A_G = -3.04 \times C_u + 52.02 \tag{8c}$$

Regarding the parameter A_D as well as the exponents n_G and n_D, there was not observed a clear trend of the effect of C_u or D_{50}. Thereupon, we decided to use average values of the aforementioned parameters, also depicted in Figs. 13 and 14. In the same figures, the variation of one standard deviation is also shown. It is concluded that the quarry quartz sands of angular particles exhibit in general higher n_G values as well as higher absolute n_D values in comparison to the natural quartz sands, whereas the volcanic soils exhibit intermediate values between the natural and the quarry sands. It is noticed that there was not observed a clear trend of the effect of particle shape on the parameter A_D, whereas the volcanic soils exhibit slightly lower A_D values in comparison to the quartz sands, which, as mentioned previously in this paper, is partially attributed to the variability in particles mineralogy and morphology between quartz and volcanic soils. The mean values of the parameters A_D, n_G, and n_D are summarized in Table 5.

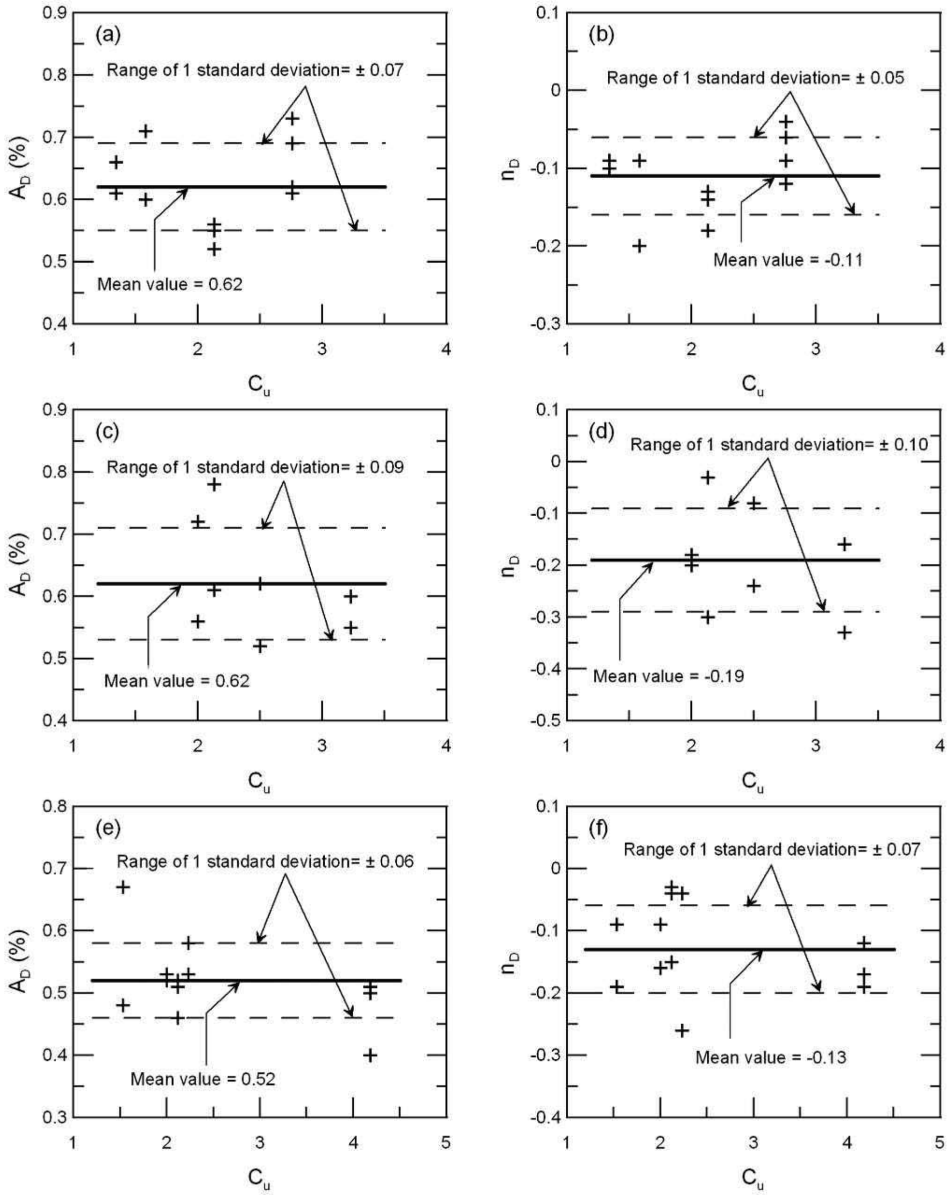

FIG. 14—*Effect of the coefficient of uniformity, C_u, on the parameter A_D and the exponent n_D of (a)–(b) natural-quartz sands, (c)–(d) quarry-quartz sands, and (e)–(f) volcanic sands.*

Thereafter, the small-strain shear modulus and damping ratio of the quartz and the volcanic sands may be estimated through Eqs 9 and 10, where G_O is given in MPa, D_O in percentile scale (%), and P_a is the atmospheric pressure. Using Eqs 9 and 10, in Fig. 15 we plot the measured against the analytically derived G_O and D_O values of the specimens of this study.

Empirical relationships for the estimation of the small-strain shear modulus of sands

$$\text{Natural quartz sands}: G_O = (-5.88 \times C_u + 57.01) \times \frac{1}{e^{0.28 \times C_u + 0.98}} \times \left(\frac{\sigma'_m}{P_a}\right)^{0.47} \tag{9a}$$

$$\text{Quarry quartz sands}: G_O = (-9.45 \times C_u + 78.15) \times \frac{1}{e^{0.28 \times C_u + 0.98}} \times \left(\frac{\sigma'_m}{P_a}\right)^{0.63} \tag{9b}$$

$$\text{Volcanic sands}: G_O = (-3.04 \times C_u + 52.02) \times \frac{1}{e^{0.28 \times C_u + 0.98}} \times \left(\frac{\sigma'_m}{P_a}\right)^{0.55} \tag{9c}$$

Empirical relationships for the estimation of the small-strain damping ratio of sands

TABLE 4—*Small-strain shear modulus and damping ratio parameters of specimens under study.*

Code name of specimen	e_o	A_G	n_G	A_D (%)	n_D
N1-1	0.661	46.3	0.49	0.60	0.09
N1-2	0.964	51.2	0.53	0.71	0.20
N2-1	0.517	37.8	0.50	0.83	0.12
N2-2	0.577	42.3	0.41	0.61	0.06
N2-3	0.588	41.5	0.49	0.69	0.09
N2-4	0.682	37.6	0.45	0.62	0.04
N3-1	0.680	44.8	0.46	0.61	0.09
N3-2	0.938	48.9	0.50	0.66	0.10
N4-1	0.566	45.9	0.45	0.52	0.14
N4-2	0.571	47.2	0.44	0.55	0.18
N4-3	0.588	46.8	0.45	0.56	0.13
Q1-1	0.683	51.3	0.53	0.56	0.18
Q1-2	0.954	56.7	0.62	0.72	0.20
Q2-1	0.545	44.2	0.63	0.55	0.16
Q2-2	0.742	47.8	0.71	0.60	0.33
Q3-1	0.594	62.5	0.63	0.78	0.03
Q3-2	0.820	61.3	0.65	0.61	0.30
Q4-1	0.553	54.2	0.57	0.52	0.08
Q4-2	0.770	60.9	0.66	0.62	0.24
V1-1	0.823	37.9	0.51	0.48	0.09
V1-2	1.083	39.1	0.57	0.67	0.19
V2-1	0.961	47.8	0.49	0.53	0.04
V2-2	1.250	47.4	0.57	0.58	0.26
V3-1	0.794	41.2	0.52	0.40	0.17
V3-2	0.898	36.8	0.55	0.51	0.12
V3-3	0.991	34.9	0.60	0.50	0.19
V4-1	0.943	50.2	0.54	0.52	0.09
V4-2	1.194	46.6	0.61	0.53	0.16
V5-1	0.976	54.1	0.55	0.46	0.03
V5-2	1.011	47.0	0.52	0.51	0.04
V5-3	1.229	48.8	0.56	0.52	0.15

$$\text{Natural quartz sands}: D_O = 0.62 \times \left(\frac{\sigma'_m}{P_a}\right)^{-0.11} \quad (10a)$$

$$\text{Quarry quartz sands}: D_O = 0.62 \times \left(\frac{\sigma'_m}{P_a}\right)^{-0.19} \quad (10b)$$

TABLE 5—*Small-strain shear modulus and damping ratio parameters (mean values).*

Soil group	n_G	A_D (%)	n_D
Natural quartz sands	0.47	0.62	−0.11
Quarry quartz sands	0.63	0.62	−0.19
Volcanic sands	0.55	0.52	−0.13

$$\text{Volcanic sands}: D_O = 0.52 \times \left(\frac{\sigma'_m}{P_a}\right)^{-0.13} \quad (10c)$$

Conclusions and Recommedations for Further Research

We presented a synthesis of low-amplitude torsional resonant column test results on reconstituted dry sands. In specific, we tested natural-quartz sands of sub-rounded to rounded particles, quarry-quartz sands of sub-angular to angular particles and volcanic granular materials composed of rhyolitic glassy rock of sub-angular to angular particles of intra-particle voids. The quarry-quartz and the volcanic sands are composed of artificially crushed rock. Both the quartz and the volcanic sands comprise geo-materials potentially used in civil engineering applications, exhibiting high shear strength, high hydraulic conductibility and, at least for low to medium confining pressures, low compressibility when prepared at high relative density of the solid matrix. The main findings of this investigation and recommendations for further research are summarized as follows:

- The volcanic sands exhibit significantly lower small-strain shear modulus (G_O) values in comparison to the quartz ones of similar grain-size distribution, prepared at the same compaction energy. This is mainly attributed to the variability in particles density, mineralogy and morphology and the overall higher void ratio and lower dry density that the volcanic sands exhibit in comparison to the quartz ones.
- The volcanic sands exhibit slightly lower small-strain damping ratio values (D_O) in comparison to the quartz sands. This is partially explained by the variability in particles mineralogy and morphology, between quartz and volcanic

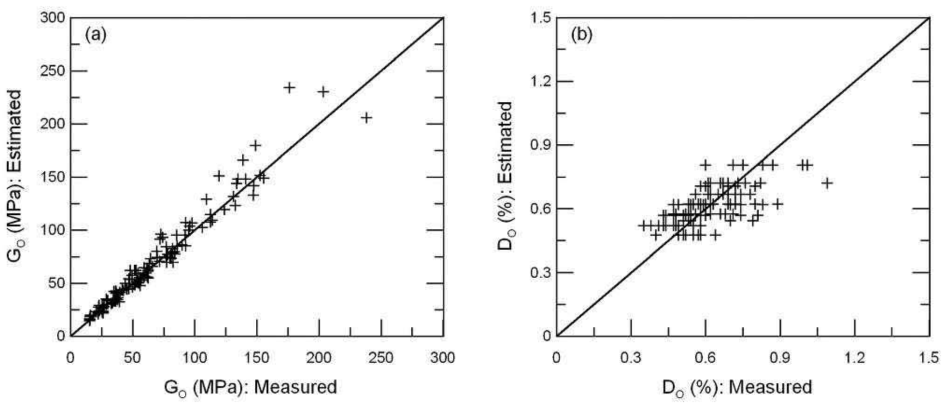

FIG. 15—*Measured against estimated values of small-strain shear modulus and small-strain damping ratio of specimens tested.*

soils. To precisely explain the observed differences in the damping ratio values between volcanic and quartz sands, it is believed that further research is needed because the mechanical properties of the volcanic particles as well as the response at particle contacts are not well known. For this reason, further research is needed including micro-scale tests where the normal and shear stiffness at particle contacts and the elastic properties of the individual particles will be examined.

– Most empirical relationships proposed in the literature for the estimation of the small-strain shear modulus and damping ratio of granular soils predominately composed of quartz sands are not valid for the volcanic soils examined in this study.
– The effects of mean effective confining pressure, void ratio and grain size distribution on the dynamic properties G_O and D_O of the volcanic sands follow a similar trend as in granular soils predominately composed of quartz particles. In specific, G_O increases with increasing σ'_m and decreasing e, whereas D_O slightly decreases with increasing σ'_m. At a given void ratio and mean effective confining pressure, G_O increases as the coefficient of uniformity decreases. However, there was not observed a monotonic trend of the effect of mean grain size on G_O, or void ratio and grain-size distribution on D_O for both the quartz and the volcanic sands. In addition, the coefficient of uniformity seems to affect in a more pronounced way the stiffness of the quartz sands in comparison to the volcanic ones.
– Particles shape affects significantly the response of the quartz sands in the region of very small strains; the increase of G_O and decrease of D_O with increasing σ'_m is more pronounced in the case of the quarry quartz sands of sub-angular to angular particles in comparison to the natural quartz sands of sub-rounded to rounded particles. This is mainly attributed to the higher deformability at particle contacts as mean roundness decreases.
– For practical-design purposes, we proposed relationships of empirical form, stemming from the resonant column test data, for the estimation of the dynamic properties of G_O and D_O separately for natural sands of sub-rounded to rounded quartz particles, quarry sands of sub-angular to angular quartz particles and volcanic sands composed of rhyolitic glassy rock of sub-angular to angular particles.
– The literature relative to the dynamic properties of volcanic soils is extremely limited and thus it is believed that further research is needed including natural volcanic soils or crushed rock of variable types, because in this study we examined the coarse fractions of a volcanic geo-material of specific mineralogy composed of sub-angular to angular particles (artificially crushed rock).
– The experimental data of this investigation will be enriched if different apparatuses are used; bender elements for further research in the region of very small strains, as well as cyclic triaxial and/or torsional shear tests for further research in a wider range of shearing strain amplitudes. At a preliminary stage, we tested herein reconstituted dry volcanic sands to examine the response of the dry solid skeleton, but this investigation may be extended with tests on fully or partly saturated specimens as well as on coarser volcanic soils.

Acknowledgments

The writers would like to thank the anonymous reviewers for their constructive comments and their detailed suggestions, which helped us to improve the quality of the paper. During the preparation and writing of this paper, the first author was supported by the Strategic Research Grant 7002667 entitled "The development of a new soil particle loading apparatus" and the Start-Up Research Grant 9380056 entitled "Experimental soil mechanics" funded by the City University of Hong Kong.

References

Anastasiadis, A., 1994, "Dynamic Properties of Typical Greek Soils," Ph.D. dissertation, Department of Civil Engineering, Aristotle University of Thessaloniki, Greece (in Greek).

ASTM D4015-92, 1992, "Standard Test Methods for Modulus and Damping of Soils by the Resonant Column Method," *Annual Book of ASTM Standards*, ASTM International, West Conshohocken, PA.

ASTM D2487-00, 2000, "Standard Practice for Classification of Soils for Engineering Purposes (Unified Soil Classification System)," *Annual Book of ASTM Standards*, ASTM International, West Conshohocken, PA.

ASTM D4254-00, 2000, "Standard Test Methods for Minimum Index Density and Unit Weight of Soils and Calculation of Relative Density," *Annual Book of ASTM Standards*, ASTM International, West Conshohocken, PA.

ASTM D854-02, 2002, "Standard Test Methods for Specific Gravity of Soil Solids by Water Pycnometer," *Annual Book of ASTM Standards*, ASTM International, West Conshohocken, PA.

ASTM D1557-02, 2002, "Standard Test Methods for Laboratory Compaction Characteristics of Soil Using Modified Effort (2,700 kN-m/m^3)," *Annual Book of ASTM Standards*, ASTM International, West Conshohocken, PA.

ASTM D3080-03, 2003, "Standard Test Methods for Direct Shear Test of Soils Under Consolidated Drained Conditions," *Annual Book of ASTM Standards*, ASTM International, West Conshohocken, PA.

Atkinson, J. and Sallfors, G., 1991, "Experimental Determination of Soil Properties," Proceedings of the 10th European Conference on Soil Mechanics, Vol. 3, Florence, Italy, pp. 915–956.

Blott, S. and Pye, K., 2008, "Particle Shape: A Review and New Methods of Characterization and Classification," *Sedimentology*, Vol. 55, No. 1, pp. 31–63.

Bui, M., Clayton, C., and Priest, J., 2007, "Effects of Particle Shape on G_{MAX} of Geomaterials," Proceedings of the 4th International Conference on Earthquake Geotechnical Engineering, Thessaloniki, Greece, June 25–28, Paper No. 1536.

Cavarretta, I., 2009, "The Influence of Particle Characteristics on the Engineering Behaviour of Granular Materials," Ph.D. dissertation, Department of Civil and Environmental Engineering, Imperial College, London, UK.

Cavarretta, I., Coop, M., and O'Sullivan, C., 2010, "The Influence of Particle Characteristics on the Behavior of Coarse Grained Soils," *Géotechnique*, Vol. 60, No. 6, pp. 413–423.

Cho, G.-C., Dodds, J., and Santamarina, C., 2006, "Particle Shape on Packing Density, Stiffness, and Strength," *J. Geotech. Geoenviron. Eng.*, Vol. 132, No. 5, pp. 591–602.

Chung, R., Yokel, F., and Drnevich, V., 1984, "Evaluation of Dynamic Properties of Sands by Resonant Column Testing," *Geotech. Testing J.*, Vol. 7, No. 2, pp. 60–69.

Clayton, C., Theron, M., and Vermuelen, N., 2004, "The Effect of Particle Shape on the Behaviours of Gold Tailing," *The Advance in Geotechnical Engineering—The Skempton Conference*, Vol. 1, J. R. Jardin, D. M. Potts, and K. G. Higgins, Eds., ICE, the Royal Geographical Society, Thomas Telford Ltd., London, UK, pp. 393–404.

Clayton C., 2011, "Stiffness at Small Strain: Research and Practice," *Géotechnique*, Vol. 61, No. 1, pp. 5–37.

Cundall, P. and Strack, O., 1979, "A Discrete Numerical Model for Granular Assemblies," *Géotechnique*, Vol. 29, pp. 49–65.

Drnevich V., 1967, "Effects of Strain History on the Dynamic Properties of Sand," Ph.D. dissertation, University of Michigan.

Edil, T. and Luh, G., 1978, "Dynamic Modulus and Damping Relationships for Sands," Proceedings of the Geotechnical Division Speciality Conference on Earthquake Engineering and Soil Dynamics, Vol. 1, ASCE, Pasadena, CA, pp. 394–409.

Gazetas, G., 1991, "Foundation Vibrations," *Foundation Engineering Handbook*, H. Y. Fang, Ed., Van Nostrand Reinhold, New York, pp. 553–593.

Hardin, B. and Richart, F., 1963, "Elastic Wave Velocities in Granural Soils," *J. Soil Mech. Found. Div.*, Vol. 89, No. SM1, pp. 33–65.

Hardin, B. and Drnevich, V., 1972a, "Shear Modulus and Damping in Soils: Measurement and Parameter Effects," *J. Soil Mech. Found. Div.*, Vol. 18, No. 6, pp. 603–624.

Hardin, B. and Drnevich, V., 1972b, "Shear Modulus and Damping in Soils: Design Equations and Curves," *J. Soil Mech. Found. Div.*, Vol. 98, No. 7, pp. 667–692.

Hardin B., 1978, "The Nature of Stress Strain Behavior of Soils," Proceedings of the Geotechnical Division Speciality Conference on Earthquake Engineering and Soil Dynamics, Vol. 1, ASCE, Pasadena, CA, pp. 3–90.

Hardin, B. and Kalinski, M., 2005, "Estimating the Shear Modulus of Gravelly Soils," *J. Geotech. Geoenviron. Eng.*, Vol. 131, No. 7, pp. 867–875.

Ishihara, K., 1996, "*Soil Behaviour in Earthquake Geotechnics*," Oxford Science Publications.

Iwasaki, T. and Tatsuoka F., 1977, "Effects of Grain Size and Grading on Dynamic Shear Moduli of Sands," *Soils Found.*, Vol. 17, No. 3, pp. 19–35.

Iwasaki, T., Tatsuoka, F., and Takagi, Y., 1978, "Shear Moduli of Sands under Cyclic Torsional Shear Loading," *Soils Found.*, Vol. 18, No. 1, pp. 39–56.

Jamiolkowski, M., Leroueil, S., and Lo Priesti, D., 1991, "Design Parameters from Theory to Practice," Proceedings of the International Conference on Geotechnical Engineering for Coastal Development: Geo-Coast 1991, Coastal Development Institute of Technology, Yokohama, Japan, pp. 877–917.

Johnston, D., 1981, "Attenuation: A State of the Art Summary," *Seismic Wave Attenuation*, M. N. Toksoz and D. H. Johnston, Eds., Society of Exploration Geophysical, Tulsa, pp. 123–135.

Kanataki, M., Nishi, K., and Tanaka, Y., 1994, "Dynamic Properties of Sand at Low Confining Pressure," Proceedings of the 1st International Conference on Pre-Failure Deformation Chracteristics of Geomaterials, Vol. 1, Sapporo, Japan, Sept. 12–14, pp. 37–40.

Kokusho, T., 1980, "Cyclic Triaxial Test of Dynamic Soil Properties for Wide Strain Range," *Soils Found.*, Vol. 20, No. 2, pp. 45–60.

Krumbein, W. and Sloss, L., 1963, "*Stratigraphy and Sedimentation*," W.H. Freeman and Company, San Francisco, CA.

Laird, J., 1994, "Linear and Nonlinear Dynamic Properties of Soil at High Confining Pressures," M.S. Dissertation, University of Texas at Austin, Austin, TX.

Lontou, P. and Nikolopoulou, C., 2004, "Effect of Grain Size on Dynamic Shear Modulus of Sands: An Experimental Investigation," University of Patras, Greece (in Greek).

Menq, F.-Y., 2003, "Dynamic Properties of Sandy and Gravelly Soils," Ph.D. Dissertation, University of Texas at Austin, Austin, TX.

Menq, F.-Y. and Stokoe, K., 2003, "Linear Dynamic Properties of Sandy and Gravelly Soils from Large-Scale Resonant Tests," *Deformation Characteristics of Geomaterials*, H. Di Benedetto, T. Doanh, H. Geoffroy, and C. Sauzeat, Eds., Swets & Zeitlinger, Lisse, pp. 63–71.

Orense, R. P., Zapanta, A., Hata, A., and Towhata, I., 2006, "Geotechnical Characteristics of Volcanic Soils Taken from Recent Eruptions," *J. Geotech. Geol. Eng.*, Vol. 24, pp. 129–161.

Pitilakis, K. and Anastasiadis, A., 1994, "Dynamic Properties of Saturated Sands Measured With R-C Device," Proceedings of the XIII ICSMFE, New Delhi, India, pp. 205–210.

Radjai, F., Wolf, D., Jean, M., and Moreau, J.-J., 1998, "Bimodal Character of Stress Transmission in Granular Packing," *Phys. Rev. Lett.*, Vol. 80, No. 1, pp. 61–64.

Richart, F., Woods, R., and Hall, J., 1970, "*Vibrations of Soils and Foundations*," Prentice-Hall, Englewood Cliffs, NJ.

Santamarina, J. and Cascante, G., 1998, "Effect of Surface Roughness on Wave Propagation Parameters," *Géotechnique*, Vol. 48, No. 1, pp. 129–137.

Santamarina, C., Klein, K., and Fam, M., 2001, "*Soils and Waves*," John Wiley and Sons, New York.

Saxena, S. and Reddy, K., 1989, "Dynamic Moduli and Damping Ratios for Monterey No. 0 Sand by Resonant Column Tests," *Soils and Foundations*, Vol. 29, No. 2, Japanese Society of Soil Mechanics and Foundation Engineering, pp. 37–51.

Senetakis, K., 2011, "Dynamic Properties of Granular Soils and Mixtures of Typical Sands and Gravels With Recycled Synthetic Materials," Ph.D. Dissertation, Department of Civil Engineering, Aristotle University of Thessaloniki, Greece (in Greek).

Senetakis, K., Anastasiadis A. and Pitilakis K., 2012a, "Dynamic Properties of Dry Sand/Rubber (RSM) and Gravel/Rubber (GRM) Mixtures in a Wide Range of Shearing Strain Amplitudes," *Soil Dyn. Earthquake Eng.*, Vol. 33, pp. 38–53.

Senetakis, K., Anastasiadis, A., Pitilakis, K., and Souli, A., 2012b, "Dynamic Behavior of Sand/Rubber Mixtures. Part II: Effect of Rubber Content on G/Go-γ-DT Curves and Volumetric Threshold Strain," *J. ASTM Int.*, Vol. 9, No. 2, ID JAI103711.

Shibata, T. and Soelarno, D., 1975, "Stress Strain Characteristics of Sands under Cyclic Loading," Proceedings of the Japanese Society of Civil Engineering, Vol. 239, pp. 57–65.

Vucetic, M., 1994, "Cyclic Threshold Shear Strains in Soils," *J. Geotech. Eng.*, Vol. 120, No. 12, pp. 2208–2228.

Wichtmann, T. and Triantafyllidis, Th., 2009, "Influence of the Grain-Size Distribution Curve of Quartz Sand on the Small Strain Shear Modulus G_{max}," *J. Geotech. Geoenviron. Eng.*, Vol. 135, No. 10, pp. 1404–1418.

Wichtmann, T., Hernandez, M., Martinez, R., Graeff, D., and Triantafyllidis, Th., 2011, "Estimation of the Small-Strain Stiffness of Granular Soils Taking into Account the Grain Size Distribution Curve," Proceedings of the 5th International Conference on Earthquake Geotechnical Engineering, Santiago, Chile.

Youd T., 1973, "Factors Controlling the Maximum and Minimum Densities of Sands," *Evaluation of Relative Density and Its Role in Geotechnical Projects Involving Cohesionless Soils*, American Society for Testing and Materials, Philadelphia, PA, pp. 98–112.

Yu, P. and Richart, F., 1984, "Stress Ratio Effects on Shear Modulus of Dry Sands," *J. Geotech. Eng.*, Vol. 110, No. 3, pp. 331–345.

Zambelli, C., Prisco, C., d'Onofrio, A., Visone, C., and Magistris, F., 2006, "Dependency of the Mechanical Behaviour of Granural Soils on Loading Frequency: Experimental Results and Constitutive Modelling," Proceedings of the Soil Stress-Strain Behavior: Measurement, Modeling and Analysis, Geotechnical Symposium, Rome, March 16–17.

Zhou, Y. and Chen, Y., 2005, "Influence of Seismic Cyclic Loading History on Small Strain Shear Modulus of Saturated Sands," *Soil Dyn. Earthquake Eng.*, Vol. 25, pp. 341–353.